DETAILED PLANNING AND INVENTORY OPTIMIZATION MODEL IN THE MATTRESS INDUSTRY

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Abstract. In this work, a mixed integer linear programming (MILP) model for the simultaneous production and inventory planning problems is proposed, which arise in the industrial context of polyurethane foam manufactory. The production process considers the foam blooming stage as well as the curing step where the blocks are dried. Several tradeoffs are involved in the overall model given that detailed inventory management decisions are also taken into account in the formulation. Several study cases provided by a local company are tested to show the model performance.

Keywords: production planning, inventory management, polyurethane foam, MILP.

1. Introduction

Mattress companies have to deal with a very competitive environment. In order to capture consumers' preference they constantly introduce new products in the market and handle a huge variety of them. Nowadays, the demand of simple foam mattresses is lower than the spring mattresses. This situation leads to more complex production planning process. In addition, given the large size and diversity of the foams required in the final products, keeping a balanced and reasonable inventory is also a challenging issue.

The problem considered in this work involves the production planning and stock management problems of foam blocks for a mattress manufacturing plant. The studied AAIQ Asociación Argentina de Ingenieros Químicos - CSPQ

process comprises three basic stages: foam production line, curing of foam blocks and stock of foam blocks. To the best of our knowledge, the literature on the problem approached in this article is scarce. One of the main characteristics is that detailed production planning and inventory decisions are optimized simultaneously. In this sense, some works integrating process and logistic decisions has been proposed by many authors and applied in several industries.

Many authors have pointed out that including different planning levels in the company decision is a challenging issue in order to obtain efficient results (Maravelias and Sung, 2009; Fumero et al., 2011; Grossmann, 2012). For instance, Rodriguez and Vecchietti (2010) integrate the inventory and delivery optimization problems under seasonal demand in the supply chain. In Fumero et al. (2013), decisions about supply chain and batch plant design are simultaneously considered, and detailed production planning is included through mixed product campaign operation mode. Two alternative MILP models are proposed by Relvas, et al. (2013) for integrated scheduling and inventory management of an oil products distribution system. The approaches aim to attain a set of planning objectives such as fulfilling customer demands while minimizing the medium flow rate and avoiding excessively low final inventory levels. In a similar industry, Marchetti, et al. (2014) propose a multi-period mixed-integer linear programming model (MILP) for the simultaneous production and distribution of industrial gas supply-chains.

Few articles deal with the production planning problem in this specific industry. Lin et al. (2013) present scheduling model inspired by a real production line of polyurethane (PU) foam at a manufacturing site in central Taiwan. In Mogaji (2014), a decision support system for process planning and control of PU foam production is proposed. The system is developed to enhance production efficiency and presents seven modules that work together to support the decision-making task. It also includes a simple linear programming (LP) model to minimize the production costs considering limited raw material in each period.

In the present work, an MILP model for solving the planning and inventory problem in a PU foam production plant is proposed. Different decision levels are involved, jointly formulated and solved in the proposed approach. The production process considers not only the foam blooming stage, but also the curing step which involves the arrangement of blocks in a limited area. Detailed inventory management decisions are also embedded in the proposed formulation, making the problem more difficult to be solved but more realistic at the same time. In this way, the presented formulation integrates several decisions and therefore various tradeoffs are simultaneously considered and evaluated.

2. Problem Description

The proposed approach is motivated by production planning and stock management problems of foam blocks for a mattress manufacturing plant. The problem considered in this work involves three basic stages: foam production line, curing of foam blocks and stock of foam blocks.

The production is carried out in a single foam blooming machine where various densities of foam can be produced. There is a set of possible foam widths but only one width can be selected per day given that a long setup machine is required between width changes. The foam is cut according to different lengths and densities, forming the blocks. There is a specific density sequence to be produced, taking into account production requirements. Also, in order to reduce scraps, when a density is produced, a minimal length must be planned.

The second stage is the curing process where the blocks need to diminish their temperature, dry and obtain the appropriate structural properties. For this purpose, the blocks are arranged in a limited area for 24 hours. They are moved from the foam machine to the assigned place by a cart. The blocks are introduced in the curing area in the same order they are processed in the foaming machine. They are placed lengthwise forming rows as shown in Figure 1. The first row is located in the back of the area where blocks are placed from left to right. A fixed space between two consecutive blocks is left in order to favor the curing process. All the blocks can be cured lying down in the floor of the curing area but some blocks of low density and short length can be also cured in standing position using a smaller surface. Taking into account the produced foam width, a maximum number of rows can be located in the curing area.

For some widths and densities, longer blocks are foamed. The length of these blocks is approximately four times longer than the others, and some of them can be cured on special carts depending on their density as shown in Fig. 1.

After 24 hours, the blocks can be piled up and stored. The inventory area is organized forming groups of blocks, which have an assigned limited capacity at the storage area. Each block is classified according to its width, length and density, and it belongs to a unique inventory group. Figure 1 shows the stages comprised in this work.

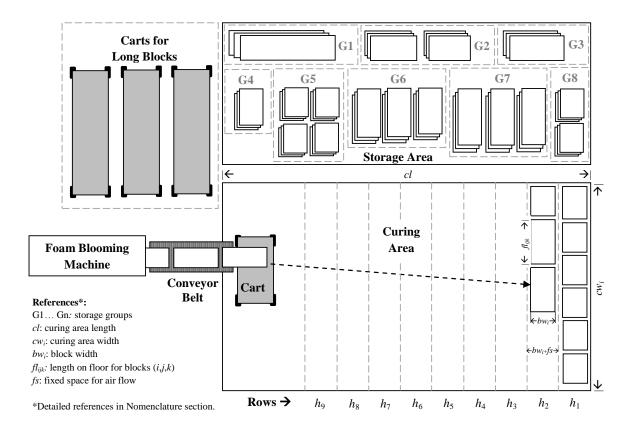


Fig. 1. Production, curing and storage of foam blocks of a mattress manufacturing plant

Every day, a known amount of blocks is taken from the storage area and they are cut for assembling the mattresses. It is important to mention that if certain block required in the downstream process is not available in storage, the production order might be postponed or another block must be used instead of the one required in the order. In the second case, this means a greater loss of material if a larger block is used or more expensive products if a block of higher density replaces the one missing. Therefore, the

main objective of the foam manufacturing area is to keep a sufficiently large and balanced stock of blocks to avoid this type of situations taking into account the available space in the storage area.

After testing different performance measures in order to find the appropriate objective function, it was detected that the firm wanted to produce as much as possible foamed blocks keeping a reasonable stock of each block. For that purpose, the difference between the inventory level and the required minimal stock is calculated and taken into account in the objective function.

Then, the problem for the simultaneous optimal production planning, curing and stock of foam blocks is stated as follows:

Given:

- the set of widths, lengths and densities of blocks
- the order of densities
- the minimum length per density
- the minimum length to be foamed in the planning horizon
- the curing area size (width and length)
- the number of carts and the set of blocks that can be cured in carts
- the set of blocks that can be cured in standing position
- the stock level for each group at the beginning of the day
- the minimum stock level for each type of block
- the demand of blocks of each type required from the downstream process
- the storage groups and the blocks that compose each group
- the capacity of each storage group (number of blocks of each type)

Determine:

- the daily width to be foamed
- the lengths and densities for the selected width
- the number of blocks of each type to be foamed
- the sequence in which the blocks are foamed
- the blocks arrangement in the curing area
- the total stock of blocks at the end of the working day

with the objective of maximizing the total foamed length minus the total length of inventory shortage, fulfilling the minimum stock requirement for the blocks of the foamed width.

3. Mathematical Model

The model basically considers: i) planning constraints, ii) stock constraints, and iii) the objective function. Due to space reasons, a brief description of the mathematical formulation is presented. The detailed formulation is not provided in this manuscript, but readers can request it to the authors.

The planning constraints involve equations related to the width to be foamed, the number of blocks for each density and length, minimum production requirements, block arrangement in the curing area and carts, and logical constraints between discrete decision variables. Stock equations consider the stock management relations and storage limitations. The minimal stock requirement must be satisfied for all blocks of the selected width. For the other widths, the concept of lacking blocks is introduced for calculating the total length of blocks missing in the inventory according to the minimal stock requirement. In this way, the objective function maximizes the total length of foamed blocks minus the lacked length of not foamed width.

The resulting formulation gives rise to a Mixed Integer Linear Programming (MILP) model. The model achieves an appropriate planning of the foaming sector with adequate links with production through inventory management. Previous efforts with disarticulated proposals failed to attain an efficient operation. The concurrent consideration of stock capacity, production requirements and curing limitations allows tighter resources utilization. Thus, mathematical programming is an effective tool to improve the production and inventory management.

The result of this model proposes a daily production plan considering process and storage requirements as well as business policies from the company.

4. Results

Different production scenarios can be obtained according to the problem input data. In this work three study cases are presented and thorough analyzed. In the last example, the objective function is tested and validated. All cases were implemented and solved in

GAMS (Rosenthal, 2013) version 24.1.3 with a 2.8 GHz Intel Core i7 processor. The CPLEX 12.5.1 solver was employed for solving the MILP problems.

For the proposed examples, 3 possible widths (190 cm, 200 cm and 214 cm), 23 different densities, and 19 lengths, are considered. Table 1 shows, for each possible foaming block belonging to the set *Blocks* grouped according to the stock groups (*Groups*), the stock capacity of each block and group, and the minimum stock for each block. Table 2 displays the initial stock and the demand for the proposed instances.

The curing area size is determined by the parameters cw_i and cl_i . In these examples, for i equal to width 190 and 200 cm the curing area width is fixed to 2040 cm width, while the curing area length 4500 cm. When width 214 cm is selected to be foamed, some long blocks are produced, and therefore more blocks can be accommodated in the curing area if this area is transposed. Then, for i equal to 214, $cw_i = 4500$ and $cl_i = 2040$ cm. It is worth mentioning that planners can modify these parameters in order to improve the blocks arrangement in this sector. The number of rows in the curing area is calculated according to the following equation

$$Rows_i \leq \frac{cl_i}{(bw_i + fs)} \quad \forall i$$

where fs is the space between blocks for air flow and bw_i is the block width (see Fig. 1). The minimum density length is 1500 cm and the considered value for the minimum diary length is 15000 cm. There are 5 available places in special carts for curing long block, i.e. blocks of 1200 cm long. In addition, 10 blocks of width 214, density BS28, and length 225 are required as special order. Therefore, if the width 214 is selected to be produced, the total special order can be manufactured.

Table 1. Stock capacity data.

Group	Block	Stock capacity	Minimum	Group stock
	(width.density.length)	(number of	stock (number	capacity (number
		blocks)	of blocks)	of blocks)
G1	200.VE22.200	8	2	23
	200.VE22.240	2	0	

	200.VE22.280	8	2		
	200.VE22.300	5	1		
G2	200.LI18.200	10	3	25	
	200.LI18.240	3	0		
	200.LI18.280	6	1		
	200.LI18.300	6	0		
G3	200.GR28.160	22	8	39	
	200.GR28.180	8	2		
	200.GR28.210	9	6		
G4	200.AZ24.160	26	10	47	_
	200.AZ24.210	15	5		
	200.AZ24.300	6	1		
G5	190.AZ24.240	11	1	30	_
	190.AZ24.280	19	10		
G6	190.AM20.240	45	10	45	_
G7	190.AM20.260	4	0	48	_
	190.AM20.280	44	20		
G8	190.GR28.260	6	0	41	_
	190.GR28.280	35	8		
G9	190.GR28.240	26	5	26	_
G10	190.BCOL.240	53	10	53	_
G11	190.N18L.240	34	10	59	_
	190.N18L.280	25	2		
G12	200.B14P.270	11	2	32	_
	200.B14P.300	21	8		
G13	190.BC35.240	2	0	15	_
	190.BC35.280	13	3		
G14	200.AM20.180	5	2	13	
	190.AM20.200	4	0		
	200.AM20.200	4	2		
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G15	200.AM20.160	30	8	30
G16	200.CO28.160	11	0	66
	190.CO28.200	27	10	
	200.CO28.210	14	0	
	190.CO28.250	5	2	
	190.CO28.270	4	1	
	190.CO28.280	5	1	
G17	200.RS26.160	12	5	20
	200.RS26.210	8	2	
G18	190.RS26.280	10	1	10
G19	190.LI18.240	15	0	29
	190.LI18.280	14	0	
G20	200.BC35.160	11	5	24
	200.BC35.210	13	2	
G21	200.N18L.160	31	8	31
G22	200.N18L.180	5	2	13
	190.N18L.200	4	0	
	200.N18L.200	4	2	
G23	214.BS28.1200	2	0	31
	214.FR28.1200	8	2	
	214.GS26.1200	6	1	
	214.BB12.1200	10	2	
	214.BB20.1200	5	2	

Table 2. Initial stock and demands.

	Initial Stock (number of		Demand (number of	
Block	bloo	eks)	blo	cks)
(width.density.length)	Example 1	Examples	Example 1	Examples
		2 and 3		2 and 3
200.VE22.200	5	5	0	2

200.VE22.240	0	2	0	0
200.VE22.280	3	5	3	2
200.VE22.300	3	5	0	2
200.LI18.200	7	10	0	2
200.LI18.240	2	3	0	2
200.LI18.280	0	0	0	0
200.LI18.300	3	2	0	2
200.GR28.160	20	11	1	10
200.GR28.180	6	6	0	6
200.GR28.210	7	7	0	7
200.AZ24.160	23	13	1	13
200.AZ24.210	5	3	4	3
200.AZ24.300	5	3	0	3
190.AZ24.240	7	7	0	0
190.AZ24.280	18	10	1	1
190.AM20.240	40	35	16	16
190.AM20.260	2	2	2	2
190.AM20.280	40	37	5	5
190.GR28.260	3	3	0	0
190.GR28.280	35	32	13	13
190.GR28.240	23	23	5	5
190.BCOL.240	30	53	0	0
190.N18L.240	29	29	4	4
190.N18L.280	20	20	6	6
200.B14P.270	9	5	1	4
200.B14P.300	19	10	3	9
190.BC35.240	0	0	0	0
190.BC35.280	12	0	2	2
214.BS28.1200	0	0	0	0
214.FR28.1200	5	5	3	3
214.GS26.1200	4	4	0	0

214.BB12.1200	2	0	0	0
214.BB20.1200	4	2	0	0
200.AM20.180	0	0	0	0
190.AM20.200	4	4	4	4
200.AM20.200	0	0	0	0
200.AM20.160	0	0	0	0
200.CO28.160	11	11	2	11
190.CO28.200	21	21	7	7
200.CO28.210	13	14	0	13
190.CO28.250	5	1	1	1
190.CO28.270	3	1	0	0
190.CO28.280	2	2	1	1
200.RS26.160	4	4	4	4
200.RS26.210	3	4	2	3
190.RS26.280	4	4	2	2
190.LI18.240	10	8	1	1
190.LI18.280	10	8	0	0
200.BC35.160	11	5	0	5
200.BC35.210	10	13	1	10
200.N18L.160	23	13	1	13
200.N18L.180	2	3	0	2
190.N18L.200	2	2	2	2
200.N18L.200	3	3	1	3
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Example 1

Besides the parameters previously presented, consider the initial stock and the demand presented in the second and fourth columns of Table 2, respectively.

The optimal solution selects to foam 152 blocks of width 190 cm and the total foamed length is equal to 392.62 m. In Figure 2, the detailed foaming program is shown. The order in which blocks for each density are produced is provided by the

company which is a model input. However, the program selects the densities to produce. From this figure, the total foamed length for each density can be calculated.

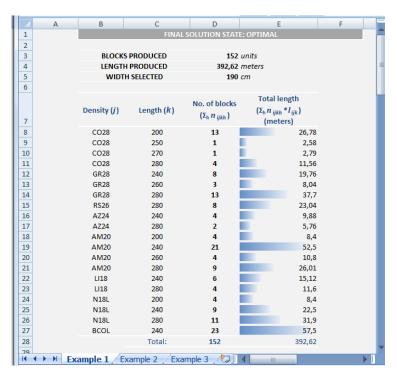


Fig. 2. Detailed program of foamed blocks.

Figure 3 displays the optimal planning in the curing area. As it can be noted, the total available rows (21) are used, but some of them are not completed in the sense that the total occupied width is less than the total available one. For example, in row h3 there are two blocks of density CO28 and length 2 m, which are lying down on the floor occupying 4.42 m of this row. Therefore, near 16 m of this row are empty. Note that the width used in this row is 4.42 meters because an additional fixed space must be considered to allow the air flow in the curing process. A similar situation can be observed in rows h7, h8, and h20. It is worth mentioning that in h20 for example, the blocks are standing, therefore the length occupied in the floor is calculated according to the block height which is 1.8 m, i.e., 6.65 meters represent 1.18 m [block height] by 5 [no. of blocks] plus 0.15 m [space between blocks] by 5 [no. of blocks]. Another feature in this case is that alternative solutions for the blocks arrangement can be obtained

because the available space in the curing area allows foaming more blocks than the available places in the storage area.

Figure 4 shows the stock management for each group. Blocks of width 190 cm are stored in groups G5 to G11, G13, G18 and G19. Groups G14, G16 and G22 include blocks of width 190 cm as well as blocks of width 200 cm. As it can be observed, some of these groups do not achieve its maximum stock capacity, even when there is free space for more blocks in the curing area. However, the free space in stock corresponds to the blocks of width 200 cm which are not produced. A similar situation occurs for groups G16 and G22. On the other hand, G13 has 5 free places for storing blocks of density BC35. However, no blocks of this density are foamed because the minimal density length cannot be satisfied, i.e., there are 2 places for blocks of length 240 cm and 3 places for blocks of length 280 cm. Then, the total possible foamed length for BC35 is equal to 1320 cm which is less than the minimal density length, equal to 1500 cm. In summary, the maximum foamed length is reached with blocks of width 190. The maximum stock capacity for this width is completed except for group G13 due to the minimum density length requirement. Therefore, the bottleneck in this case is given by the stock capacity.

The model comprises 150108 equation, 15179 continuous variables and 14907 discrete variables, and the optimal solution with a 0% optimality gap was obtained in 66.08 CPU sec.

Example 2

Considering the data shown in the third and fifth columns of Table 2, the optimal solution selects to foam blocks of width 214 cm. The number of produced blocks is 38 and the total length is equal to 358.50 m. The detailed foaming program is shown in Figure 5. As it can be observed, the special order of 10 blocks of density BS28 and length 225 cm is carried out.

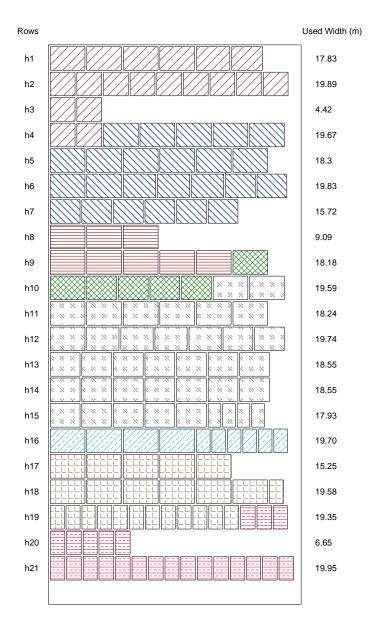


Fig. 3. Blocks arrangement in curing area.

Figure 6 shows the blocks arrangement in the curing area. As it can be noted, the number of rows in the curing area is less than in the previous case since when width 214 cm is selected, the area dimensions are transposed. In this case, all the available rows are employed and 23 long blocks are cured on the floor. Although some rows are not totally covered, no more long blocks can be accommodated in the rows of this area. Therefore, the total available space in carts (5 blocks) is used for curing long blocks of low density.

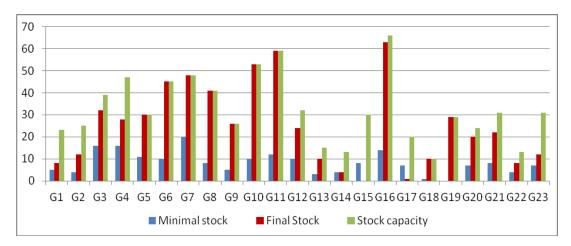


Fig. 4. Stock management of groups for Example 1.

	A	В	С	D	E	F
1	**			OLUTION STATE:		
2			,,			
3		BLOCK	S PRODUCED	38 (units	
4		LENGT	H PRODUCED	358,50		
5		WIDT	'H SELECTED	214	c m	
6						
		Density (j)	Length (K)	No. of blocks $(\Sigma_h n_{ijkh})$	Total length $(\Sigma_h n_{ijkh} * l_{ijk})$	
7		DC20	225	· · · · ·	(meters)	
8		BS28	225	10	22,50	
9		BS28	1200	2	24	
LO		FR28	1200	6	72	
11		GS26	1200	6	72	
12		BB20	1200	5	60	
13		BB12	1200	9	108	
14			Total:	38	358,50	
15						
\longleftrightarrow	► Examp	le 1 Example	2 Example 3	1)

Fig. 5. Detailed foaming program for Example 2

The stock management of these blocks is shown in Figure 7. The long blocks are grouped in a last cluster, G23. At this point, it is worth mentioning that the blocks of the special order, are not stored. There are 3 places unoccupied in G23. This means that, in this case, the most limited resource is the curing area. All the required shorter blocks are foamed, while no more long blocks are produced because they cannot be accommodated in the curing area. For the remaining groups no blocks are produced, therefore the final stock is equal to the initial stock minus the demand.

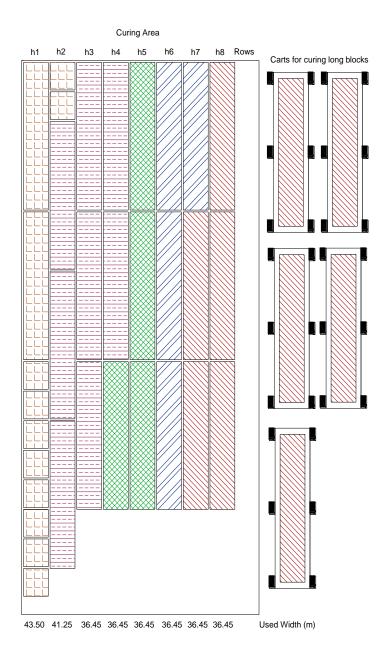


Fig. 6. Optimal blocks arrangement in the curing area for Example 2.

The optimal solution of this example was obtained in 10 CPU sec with 0% optimality gap. The model size is the same as in the previous example.

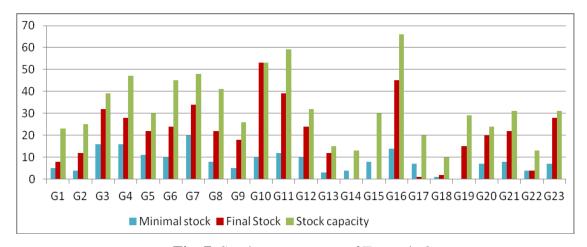


Fig. 7. Stock management of Example 2.

Example 3

The objective function is changed in this example. Now the lacked blocks according to the minimum stock are not considered. In this way, the objective function maximizes the total foamed length without taking into account the required minimum stock level. The adopted parameters in this case are those considered in Example 2. For this new objective function, the optimal solution selects width 190 cm instead of 214 cm. The detailed planning program is shown in Figure 8.

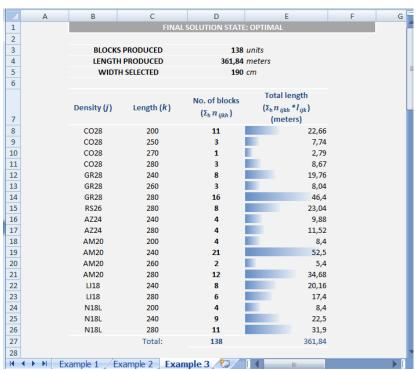


Fig. 8. Detailed production planning for Example 3

The total foamed length in the previous example was 358.5 m., while in this case, the total foamed length is 361.84 m. In order to compare both solutions, an analysis of the problem data is shown in Figure 9. From this chart, it can be noted that the shortage of blocks when width 190 is produced (point (4) in the figure), is equal to 133.43 m, while when 214 m is produced is equal to 49.43 m. Therefore, when the length of missing blocks is subtracted from the length of foamed blocks, the best solution is to produce blocks of width 214 m, while when shortage is not taken into account the best solution is to produce blocks of width 190 m.

This shows that the purpose of maintaining a more balanced stock level can be achieved when the objective function considers the missing blocks regarding to the minimal stock.

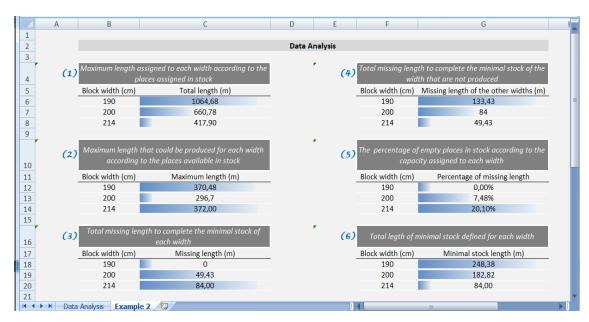


Fig. 9. Data Analysis for Examples 2 and 3.

6. Conclusion

In this article, an MILP formulation for the simultaneous optimization of PU foam production planning and inventory management was proposed. Planning decisions include two main features of the production process. In the foaming step, only one foam width can be produced per day and the number and type of foam blocks must be

decided. In the curing process, the blocks must be placed according to the available curing space. Given that only one width can be produced per day, storage planning is critical in the plant. The lack of blocks in stock is then incorporated in the objective function allowing their homogeneous distribution in storage. Therefore, the model allows obtaining the largest foaming production considering the lack of non manufactured blocks and satisfying production, curing and stock company policies.

Several trade-offs are simultaneously considered. First the model achieves an appropriate planning of the foaming sector with adequate links with production through inventory management. Previous efforts with disarticulated proposals failed to attain an efficient operation. Blocks production not only is limited production requirements but also by curing area and stock space. The concurrent consideration of these perspectives allows tighter resources utilization. Thus, mathematical programming is an effective tool to improve the production and inventory management.

The inventory management through blocks groups with limited capacity has enabled a stock policy avoiding shortages. If additional space was available in the inventory area, a dedicated stock for each block could be adopted. However, space limitations forced to adopt a different approach. Blocks groups are sufficiently flexible to maintain an appropriate blocks stock in the available area.

Three study cases were presented in order to show and test model performance. Through the examples, the approach capabilities were highlighted and several tradeoffs among production and curing processes, and stock management were analyzed. For the company, the presented approach represents a very useful tool for deciding the diary production planning.

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