Cooperative n-personal games in the coalitional economic control of a microgrid community

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***Abstract*—The deployment of microgrids connected to an electricity grid is increasing every day. These energy districts with their control system are the intelligent nodes of future electricity grids; therefore, strategies for managing these new systems must be developed and proposed. This paper presents a novel coalitional economic model predictive control strategy for managing a microgrid community. Because coalitional control considers the dynamic variation of coalitions of agents, a cooperative n-personal game with economic aspects occurs to decide which coalition to build, where the optimal control strategy to solve for each of these coalitions also takes place. Furthermore, an example proposes an economic criterion for using both the cooperative game and the control strategy for each coalition. Finally, some results on coalition formation are presented for the example mentioned above.**

***Index Terms*—Cooperative n-personal games, Game theory, Value of Shapley, Coalitional control, Model predictive control.**

I. Introduction

The future electricity system worldwide is tending towards a paradigm change in the structure of its electricity grids. This restructuring refers to moving from unidirectional, centralised schemes, with large-scale generation centres where the roles of electricity production and consumption are marked, to a decentralised one, with bidirectional characteristics for energy trade, where users lose their passive character of simple consumers, highlighting the use of renewable resources and small-scale systems for a generation.

This new grid, referred to as a smart grid, can be defined as a power grid composed of intelligent nodes which can operate, communicate and interact autonomously in the efficient distribution of electricity resources to their consumers. In this definition, one can intuit the heterogeneous nature of the smart grid, which motivates the adoption of advanced techniques to address the various technical challenges at different levels, such as the design, control, and implementation stages.

The intelligent nodes that make up this grid refer to energy entities with the capacity for auto-control and management, known as microgrids, which constitute a powerful tool to facilitate the insertion of renewable resources as a source of distributed generation for obtaining electricity, as they have storage systems and the necessary structure to deal with the explicit problems caused by the randomness of these resources.

Solving the problem of control and management of these smart grids through centralised strategies does not seem to be the best option from an implementation perspective, mainly due to the heterogeneity of the system and the particular interests of each microgrid.

A distributed control approach that considers the formation and dynamic variation of coalitions of agents with the objective of global performance, but without losing sight of the particular management aspects of each agent, emerges as an attractive control alternative. This strategy, which considers a dynamic variation in the coalitions, is known as coalitional control.

In terms of deciding which coalition option will prove to be the best choice for the management of the system, some form of tool or theoretical framework is needed with which to analyse the possible coalitions to be formed. In this respect, Game Theory offers a suitable conceptual approach.

The contribution of this work is to extend the economic model predictive control for microgrid management connected to the electrical grid proposed in [1] to a microgrid community through a coalitional approach and to use the branch of game theory (dealing with cooperative games) to analyse and decide the optimal coalition to form.

There is no abundance of works addressing the control of a microgrid community with optimal control approaches and game theory, where some of them are available in [2], [3] and [4]. In [2], the authors propose a strategy based on MPC and cooperative games; however, the control objective does not consider the system's dynamic performance, and the resulting implementation proposal is very demanding from the viewpoint of the computational burden. While in [3] and [4], the problem of system division is solved through game theory, the system dynamics are not considered, nor is it specified how the coalitions formed would determine their control actions.

The rest of the document is organized as follows: in Section

II, the problem formulation is introduced; in Section III a solution is proposed through a hierarchical coalitional control strategy; in Section IV an example is presented, while in Section V, some conclusions are drawn.

II. Problem formulation

Systems formed by groups of agents, where the possibility of open communication and interaction between any of them is considered, as a community of microgrids willing to cooperate (mesh topology [5]), can be represented using undirected graphs $\mathcal{G} = (\mathcal{N}, \gamma)$, where $\mathcal{N} \subset \mathbb{N}\backslash\{0\}$ corresponds to a finite set of agents that form the system, whereas γ is the set of all pairs of agents connected one, i.e. $\gamma = \{\{p, q\} \mid p, q \in \mathcal{N}, \, p \neq q, \text{where } p \text{ is connected with } q\}$. All $p, q \in \mathcal{N}$ are agents, and every pair $\{p, q\} \in \gamma$ is a link indicating that the agents p and q are directly connected by the graph $\mathcal{G} = (\mathcal{N}, \gamma)$.

We also define that for a given agent p, it has a set of neighbouring agents $q \in \mathcal{N}_p \subset \mathcal{N}$ with which it can communicate directly through the graph, that is to say, that $\mathcal{N}_p = \{q \mid \{p, q\} \in \gamma\}$ refers to the set of neighbouring agents to p, with which it can interact without any intermediary. When considering complete systems, which means that any pair of agents $p, q \in \mathcal{N}$ have a direct connection by the graph $\mathcal{G} = (\mathcal{N}, \gamma)$, this set will be precisely the rest of the agents that compose the system.

Since coalitional control considers the dynamic variation of groups or coalitions of agents in a system represented by a graph [6], it must be taken into account that for a given instant of time k, any given link $\{p, q\} \in \gamma$ can be enabled or disabled. Based on this, we define the concept of *Network Topology* $\Lambda_k \subseteq \gamma$, which refers to the enabled links at a specific time.

In a community of microgrids, it means that when a link is enabled, there can be an energy exchange between the microgrids connected by it. To represent this situation, we define in a microgrid p the variable $z_k^q \in \mathbb{R}\backslash\{0\} \;\; \forall\, q \in \mathcal{N}_p$ which materialises the energy exchange between the microgrids p and q. If $z_k^q \in \mathbb{R}^-\backslash\{0\}$, it means that the energy transfer occurs from the microgrid p to q ($p \rightsquigarrow q$), otherwise if $z_k^q \in \mathbb{R}^+\backslash\{0\}$, the energy goes from q to the microgrid p ($q \rightsquigarrow p$). Each microgrid (agent) $p \in \mathcal{N}$ has a local controller, where its action produces a dynamic for the local system that is represented by a discrete-time state-space model as shown below:

$$x_{k+1}^p = A^p \, x_k^p + B^p \, u_k^p \tag{1a}$$

$$B_u^p \, u_k^p + M^q \, z_k^q + E_w^p \, w_k^p = 0, \forall\, p \in \mathcal{N}\,,\, \forall\, q \in \mathcal{N}_p \tag{1b}$$

where the state variables $x_k^p \in \mathbb{R}^{n_{xp}}$ represent the load levels for the existing storage systems in the microgrid p; $u_k^p \in \mathbb{R}^{n_{up}}$ and $z_k^q \in \mathbb{R}^{n_q}$ are the manipulated variables or control actions, where z_k^q exclusively indicates the energy exchanged with neighbouring microgrids; while $w_k^p \in \mathbb{R}^{n_{wp}}$ are the non-manipulated variables or disturbance existing on the system. The matrices $A^p \in \mathbb{R}^{n_{xp} \times n_{xp}}$ and $B^p \in \mathbb{R}^{n_{up} \times n_{xp}}$, with their appropriate dimensions are the transition and inputs matrices, respectively. Whereas $B_u^p \in \mathbb{R}^{n_{up} \times 1}$, $M^q \in \mathbb{R}^{n_q \times 1}$ and $E_w^p \in \mathbb{R}^{n_{wp} \times 1}$, are matrices intended to consider the efficiency for the power converters associated with each of the variables involved in the model.

Eq. (1a) is used to describe the dynamics for storage systems, where $x_k^p \in \mathbb{R}^{n_{xp}}$ is the state vector at the current time and $x_{k+1}^p \in \mathbb{R}^{n_{xp}}$ represents the state at the next considered instant. Eq. (1b) indicates the power balance formulation to satisfy the bus or node of each microgrid.

Depending on the network topology Λ_k activated at a time instant k, which defines itself through the enabled variables z_k^q (links), the $\mathcal{C}_l \subseteq \mathcal{N}$ coalitions of microgrids are formed. To achieve coordination among the members of a coalition, a vector of manipulated variables $u_k^{\mathcal{C}_l} = (u_k^p)_{p \in \mathcal{C}_l}$ is defined, which results from arranging in a single column vector all the u_k^p corresponding to each microgrid of the coalition. Where furthermore, $z_k^{\mathcal{C}_l} = (z_k^q)_{q \in \mathcal{C}_l}$ represents the manipulated variables that symbolize the enabled links between the microgrids to form the coalition.

Once a coalition is formed, the dynamics for the system at the coalitional level will have a form analogous to Eqs. (1a) and (1b), therefore it is defined that:

$$x_{k+1}^{\mathcal{C}_l} = A^{\mathcal{C}_l} \, x_k^{\mathcal{C}_l} + B^{\mathcal{C}_l} \, u_k^{\mathcal{C}_l} \tag{2a}$$

$$B_u^{\mathcal{C}_l} \, u_k^{\mathcal{C}_l} + M^{\mathcal{C}_l} \, z_k^{\mathcal{C}_l} + E_w^{\mathcal{C}_l} \, w_k^{\mathcal{C}_l} = 0, \forall\, \mathcal{C}_l \subseteq \mathcal{N} \tag{2b}$$

where $x_k^{\mathcal{C}_l} = (x_k^p)_{p \in \mathcal{C}_l}$ and $w_k^{\mathcal{C}_l} = (w_k^p)_{p \in \mathcal{C}_l}$, refer to the vector of states and disturbance for the coalition. In the same way, as for the control actions, we order in a single column vector the states (x_k^p) and perturbations (w_k^p) of each microgrid belonging to the coalition.

Remark 1: The network topology Λ_k activated at a time instant k, divides the set of agents (microgrids) $\mathcal{N}$ into different coalitions, which is denoted as $\mathcal{P}\,(\mathcal{N}, \Lambda_k) = \{\mathcal{C}_1, \mathcal{C}_2, ..., \mathcal{C}_l\}$, where the number of coalitions is $l \in [1, |\mathcal{N}|]$, where $|\mathcal{N}|$ is the cardinality of set of agents. Here $\mathcal{P}\,(\mathcal{N}, \Lambda_k)$ is defined as a *Coalition Structure* and divides the system formed by $\mathcal{N}$ agents into several coalitions that are disjoint, verifying that: $\mathcal{C}_r \cap \mathcal{C}_s = \emptyset\, \forall\, (r, s) : r, s = 1, 2, 3, ..., l\,;\, r \neq s$ and $\cup_{j=1}^{l} \mathcal{C}_j = \mathcal{N}$.

Remark 2: The set of all coalition structures is denoted by $\mathcal{P}^{|\mathcal{N}|}$ and the set for coalition structures containing exactly (s) coalitions as $\mathcal{P}_s^{|\mathcal{N}|}$. Thereby, the number of all possible coalition structures is determined by the "*Bell Number* $B_{|\mathcal{N}|}$":

A. Definition of the optimal control problem

To control in an optimal perspective a system formed by a community of microgrids through an approach based on the formation and dynamic variation of coalitions to coordinate their control actions jointly, as cooperation between local controllers translates into an improvement of their overall performance [7], the control strategy must meet the following objectives: (i) establishes criteria and a methodology to determine the network topology Λ_k and to be able to form coalitions $\mathcal{C}_l$, or select them directly from a set of possibilities and thus define the network topology, and (ii) generate the manipulated variables for each coalition by solving an optimal control problem.

We propose to achieve the above control objectives using an MPC strategy whose associated optimisation problem takes the following form:

$$\min_{\boldsymbol{u}^p,\, \boldsymbol{z}^q(\Lambda_k)} \quad \sum_{p\in\mathcal{N}} \sum_{i=0}^{N-1} V_N^p\left(x_{i|k}^p, c_{e,k}; \boldsymbol{u}^p, \boldsymbol{z}^q\left(\Lambda_k\right)\right) \tag{3a}$$

$$\text{s.t.: } x_{0|k}^p = x_k^p, \tag{3b}$$

$$\Lambda_{0|k} = \Lambda_k,\ \Lambda_{i|k} \subseteq \gamma \tag{3c}$$

$$x_{i+1|k}^p = A^p\, x_{i|k}^p + B^p\, u_{i|k}^p, \tag{3d}$$

$$B_u^p\, u_{i|k}^p + M^q\, z_{i|k}^q + E_w^p\, w_{i|k}^p = 0, \tag{3e}$$

$$x_{i|k}^p \in \mathbb{X}^p,\ u_{i|k}^p \in \mathbb{U}^p,\ z_{i|k}^q \in \mathbb{Z}^q, \tag{3f}$$

$$x_{N-1|k}^p = x_{N|k}^p,\ i \in \mathbb{I}_{0:N-1},\ \forall\, p \in \mathcal{N},\ \forall\, q \in \mathcal{N}_p \tag{3g}$$

The optimisation variables for the problem (3) is the vector of control actions $\boldsymbol{u}^p$ defined for each microgrid and the variable $\boldsymbol{z}^q\left(\Lambda_k\right)$, which indicates the network topology to be implemented and determines the coalitions to be formed for each control instant k. It is important to note that for the variable $\boldsymbol{z}^q\left(\Lambda_k\right)$, not only the state of the link until the instant $k+1$ (enabled or disabled) is decided, but also with which values the interaction with the other agents (microgrids) will occur when it is enabled, i.e. the amount of energy to be exchanged with the neighbouring connected microgrids.

On the other hand, the optimisation parameters will be the vector of states x_k^p of the system about each microgrid and an economic index $c_{e,k}$ associated with a criterion to be optimised, which refers to the price of energy considered, to carry out the various commercial transactions.

The cost functional (3a) consists, on the one hand, of functions that seek to capture a given economic criterion (ℓ_e^p) that must be optimised for each microgrid and of the dynamic control performance, depending on the variables that define the system. On the other hand, this cost will have an appropriate formulation so that, together with the terminal constraint (3g), the optimisation problem does not lose feasibility in the face of possible changes in the economic criterion that may occur during the management of the system, as suggested in [1]. Given this, the proposed cost function is as follows:

$$\begin{aligned} &V_N^p\left(x_k^p, c_{e,k}; u_k^p, z_k^q\right) = \\ &\ell_e^p\big(x_k^p - x_{N-1}^p + x_s^p, u_k^p - u_{N-1}^p + u_s^p, z_k^q - z_{N-1}^q + z_s^q, c_{e,k}\big) \\ &+ \left\|x_k^p - x_{N-1}^p\right\|_Q^2 + \left\|u_k^p - u_{N-1}^p\right\|_R^2 + \left\|z_k^q - z_{N-1}^q\right\|_M^2 \\ &+ V_O\left(x_{N-1}^p\right),\ \forall\, p \in N,\ \forall\, q \in \mathcal{N}_p \end{aligned} \tag{4}$$

Concerning the constraints of the problem: the (3b) refers to the feedback of states at each sampling time k; while (3c) indicates the initial network topology enabled at the same control instant, as well as that any possible future enabling topology contained by the set of available links. It can also indicate as the activated coalition structure $\mathcal{P}_k\left(\mathcal{N}, \Lambda_k\right)$, which must be one of the feasible set determined by the total number of possible structures $\mathcal{P}^{|\mathcal{N}|}$, i.e. $\mathcal{P}_k\left(\mathcal{N}, \Lambda_k\right) \in \mathcal{P}^{|\mathcal{N}|}$.

The prediction model for each microgrid p is defined by the constraints (3d) and (3e), according to the set of Eqs. (1a) and (1b); while (3f) indicates that the states of each microgrid x_k^p are restricted to exist in convex sets $\mathbb{X}^p$ and that the manipulated variables u_k^p and z_k^q convex and bounded sets $\mathbb{U}^p$ and $\mathbb{Z}^q$, respectively.

However, this optimisation problem is impractical and very demanding because of the computational costs required to implement it. If we add to this the fact that it involves a control strategy executable in real-time, this difficulty becomes even more notorious. This complexity stems mainly from a variable in the optimisation problem $z_k^q\left(\Lambda_k\right)$. Because of this difficulty, it is necessary to propose a strategy that manages to relax this global problem (3), but without losing sight of the fundamental and essential objectives pursued by coalitional control, which is the dynamic division of the system under its control, to achieve the formation of coalitions of agents, and to improve performance about a distributed strategy.

III. Hierarchical coalitional control strategy

To reduce the computational complexity for the subsequent implementation of the coalitional control strategy dictated by the solution of the problem (3), a relaxed version needs to be developed. To this end, we propose to divide it into two sub-problems, each with its solution guideline, marked by the objectives (i) and (ii) established in the previous section.

The first sub-problem will determine the coalitions to form between the system's agents according to an established criterion. Once the decision has been made based on the configuration chosen for the coalitions, another sub-problem will generate the optimal control variables for each coalition, cooperatively coordinating their decisions without communicating with the agents that do not belong to the same coalition.

To achieve the necessary synchronism between each of the sub-problems, given that the solution of the first imposes conditions on the characteristics of the second, it is proposed to use a hierarchical structure composed of two levels, where each level corresponds to a particular sub-problem.

As in any hierarchical structure, each level will feature different time frames. The coalitions to form are decided periodically at the upper level on the control times used for the lower level. Therefore, if T_d indicates the sampling time to generate the control variables for each coalition (lower level), then the time where the coalitions to form (upper level) are decided will be $T_t = b\, T_d$, where $b \in \mathbb{N}\backslash\{0\}$. A similar strategy proposed to reduce the computational burden is found in [8], where such a strategy takes the form of a *top-down* approach. A representation made on a discrete timeline, as shown in Figure 1, shows the implementation of the proposed hierarchical strategy for coalitional control. Note that in this example is verified that $T_t = 5\, T_d$, resulting in $b = 5$, which is a design parameter for the hierarchical structure, where in addition, $T_d = k$ which is the sampling time for the discrete system.

The following is the treatment and formulation for each level, referred to as: (i) Determining the coalition structure and (ii) Optimal control actions at the coalition level.

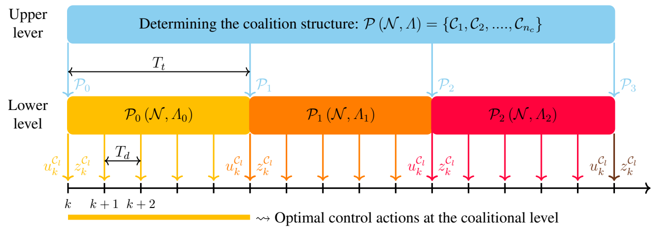

Figure 1. Hierarchical strategy for the implementation of coalitional control.

A. Determining the coalition structure

To determine the coalition structure to be implemented in the system, it is proposed to use the conceptual framework developed in [9] as a tool for analysis and decision-making, which, using a series of mathematical basics, offers the possibility of studying complex interactions between rational and independent players (agents).

Under the assumption that the different agents that form the system can communicate and exchange information between any of them, the branch of game theory dealing with cooperative n-personal games takes place. Through this, it is possible to investigate when it is convenient or advantageous for an entity to make decisions independently or to jointly act with other entities to improve its situation concerning an established criterion of interest, such as economic benefit or cost.

By using the theory of cooperative n-personal games, both the global objectives involving a system formed by several independent entities (players) come under analysis, but without losing sight of the particular, different and selfish interests that each agent may have, obtaining consensual solutions where the distinct and varied situations are analysed; logically with a marked dependence on what defines the game that is going to take place.

Remark 3: The central idea proposed to solve the problem of determining coalition structures is to use a cooperative n-personal game as a decision tool in conjunction with an MPC strategy to obtain the necessary information to choose the optimal coalitions to form between the different agents.

Using these cooperative games, the possible situations and complex transactions that can occur between the different players involved are contemplated and using the MPC strategy, the play to dictate between them is defined, where at the same time, optimal solutions are generated, allowing to consider nominal predictions for the disturbance, such as the levels of renewable generation and energy demand produced by loads of a microgrid, which have a relevant importance in the management of this type of systems.

The first reference to cooperative n-personal games appeared in [10], the definition of which follows:

Definition 1: Have a finite set of players (agents) $\mathcal{N}$ and coalitions of them $\mathcal{C} \subseteq \mathcal{N}$. A *cooperative n-personal game* in the form of a characteristic function, or Transferable Utility Game (TU-Games), is described by the pair $(\mathcal{N}, \boldsymbol{v})$, where $\boldsymbol{v}$ denotes a function that associates to each possible coalition a real number, which represents the value that the coalition obtains if its members cooperate. A coalition is any element of $2^{\mathcal{N}} = \{\mathcal{C} | \mathcal{C} \subseteq \mathcal{N}\}$, so that $\boldsymbol{v} : 2^{\mathcal{N}} \to \mathbb{R}$, where the null value is verified for the empty set $\boldsymbol{v}(\emptyset) = 0$.

As the situation modelled by a transferable utility game has a cooperative approach, the main objective in implementing these games is the distribution of the total value of the system among the different players. Thus arise the rules of sharing or allocation of each player to refer to the set solutions or solution concepts for cooperative n-personal games. The set solutions are the sharing rules, which must fulfil certain restrictions agreed upon by the players to be accepted by all of them.

When a set solution has the property that, regardless of the set, there is always a single sharing rule that satisfies all the fixed properties, it is called a singular solution, the best known and most widely applied to be the *Shapley Value* [11], which is the one used in this work.

Definition 2: The *Shapley Value* is the unique solution for every cooperative n-personal game $(\mathcal{N}, \boldsymbol{v})$ and for every player $r \in \mathcal{N}$, which results as the weighted sum of their marginal contributions:

$$\phi_r(\mathcal{N}, \boldsymbol{v}) = \sum_{\mathcal{C} \subseteq \mathcal{N} \setminus \{r\}} \frac{\mathcal{C}!\,(\mathcal{N} - \mathcal{C} - 1)!}{\mathcal{N}!} \left[\boldsymbol{v}(\mathcal{C} \cup \{r\}) - \boldsymbol{v}(\mathcal{C})\right]$$

where $\phi_r(\mathcal{N}, \boldsymbol{v}) \in \mathbb{R}$ and $\phi(\mathcal{N}, \boldsymbol{v}) = [\phi_1, \phi_2, ..., \phi_{\mathcal{N}}]^T \in \mathbb{R}^{\mathcal{N}}$.

The Shapley Value ϕ_r, as a sharing rule, satisfies the properties of: *additivity, symmetry, efficiency and null player*.

1) Definition of the characteristic function for a cooperative game: Based on the characteristic function $(\boldsymbol{v})$, we determine the guidelines of the cooperative n-personal game to be played by the players (microgrids). The domain of this function is all the possible coalitions that may form with the agents, which is defined by $2^{\mathcal{N}}$. To each coalition, the characteristic function assigns a *value*, depending on the characteristics and way of defining it (representing a cost or benefit).

The function proposed to define the cooperative game to take place between the community's microgrids will be associated with an economic criterion determined by the negotiations referring to the transfer of electrical energy that can be carried out in the different scenarios and actors in the system, depending on the price of the energy considered. This transfer may occur: (i) between each microgrid and the electricity grid and (ii) between microgrids of the same coalition.

The function is defined through an MPC strategy optimisation problem. The objective of proposing this form of definition is to consider the nominal predictions of renewable resources and energy demand, which are the no-manipulated variables in microgrid systems, where their consideration turns out to be very important when deciding on control and management actions.

The optimisation problem takes the form of Eq. (5), where the cost function $\ell_{\mathcal{C}_l}$ refers to the economic criterion to optimise, while the characteristic function for the cooperative game $\boldsymbol{v}(\mathcal{C}_l)$ indicates the cost generated for the entire prediction horizon for a particular coalition of microgrids. It should be noted that here it is appropriate to refer to it as the cost of the coalition.

$$\boldsymbol{v}\left(\mathcal{C}_l\right) = \min_{\boldsymbol{u}^{\mathcal{C}_l},\, \boldsymbol{z}^{\mathcal{C}_l}} \sum_{i=0}^{N-1} \ell_{\mathcal{C}_l}\left(x_{i|k}^{\mathcal{C}_l}, c_{e,k}; \boldsymbol{u}^{\mathcal{C}_l}, \boldsymbol{z}^{\mathcal{C}_l}\right) \tag{5a}$$

$$\text{s.t.: } x_{0|k}^{\mathcal{C}_l} = x_k^{\mathcal{C}_l}, \tag{5b}$$

$$x_{i+1|k}^{\mathcal{C}_l} = A^{\mathcal{C}_l}\, x_{i|k}^{\mathcal{C}_l} + B^{\mathcal{C}_l}\, u_{i|k}^{\mathcal{C}_l}, \tag{5c}$$

$$B_u^{\mathcal{C}_l}\, u_{i|k}^{\mathcal{C}_l} + M^{\mathcal{C}_l}\, z_{i|k}^{\mathcal{C}_l} + E_w^{\mathcal{C}_l}\, w_{i|k}^{\mathcal{C}_l} = 0, \tag{5d}$$

$$x_{i|k}^{\mathcal{C}_l} \in \mathbb{X}^{\mathcal{C}_l},\ u_{i|k}^{\mathcal{C}_l} \in \mathbb{U}^{\mathcal{C}_l},\ z_{i|k}^{\mathcal{C}_l} \in \mathbb{Z}^{\mathcal{C}_l}, \tag{5e}$$

$$i \in \mathbb{I}_{0:N-1},\ \forall \mathcal{C}_l \in \mathcal{N}.$$

2) Criteria for deciding on coalition structure: To determine which coalitions to form at each time instant defined by T_t, i.e. which coalition structure $\mathcal{P}\left(\mathcal{N}, \Lambda\right)$ is selected, we propose to use the *Shapley Value* as the solution set for the cooperative n-personal game defined for the microgrid group $\mathcal{N}$ and the characteristic function $\boldsymbol{v}$ of the problem (5).

The Shapley Value for a player r is the weighted sum of his marginal contributions, where the individual marginal contribution of player r represents the change in value that a given coalition experiences after this player joins.

The criterion proposed to select the coalition structure $\mathcal{P}\left(\mathcal{N}, \Lambda\right)$, will be the one that represents the lowest cost, based on the sum of the individual marginal contributions for all the microgrids (players) of the community, which generate when forming the coalitions defined for a specific structure.

Proposition 1: Consider two coalition structures for the same group of agents $\mathcal{N}$, i.e. $\mathcal{P}_a\left(\mathcal{N}, \Lambda_a\right) = \{\mathcal{C}_1, \mathcal{C}_2, ..., \mathcal{C}_n\}$ and $\mathcal{P}_b\left(\mathcal{N}, \Lambda_b\right) = \{\mathcal{S}_1, \mathcal{S}_2, ..., \mathcal{S}_m\}$. The structure $\mathcal{P}_a\left(\mathcal{N}, \Lambda_a\right)$ is preferred over $\mathcal{P}_b\left(\mathcal{N}, \Lambda_b\right)$, if and only if, the sum of the individual marginal contributions of all its agents to achieve to form the coalitions that define it, is strictly smaller. Therefore:

$$\mathcal{P}_a\left(\mathcal{N}, \Lambda_a\right) \to \mathcal{P}_b\left(\mathcal{N}, \Lambda_b\right) \iff$$
$$\sum_{r=1}^{\mathcal{N}} \frac{\mathcal{C}!\left(\mathcal{N} - \mathcal{C} - 1\right)!}{\mathcal{N}!}\left[\boldsymbol{v}\left(\mathcal{C} \cup \{r\}\right) - \boldsymbol{v}\left(\mathcal{C}\right)\right] <$$
$$\sum_{r=1}^{\mathcal{N}} \frac{\mathcal{S}!\left(\mathcal{N} - \mathcal{S} - 1\right)!}{\mathcal{N}!}\left[\boldsymbol{v}\left(\mathcal{S} \cup \{r\}\right) - \boldsymbol{v}\left(\mathcal{S}\right)\right]$$

where $\mathcal{C} \cup \{r\} \in \mathcal{P}_a\left(\mathcal{N}, \Lambda_a\right)$ and $\mathcal{S} \cup \{r\} \in \mathcal{P}_b\left(\mathcal{N}, \Lambda_b\right)$.

B. Optimal control actions at the coalition level

Once the coalition structure $\mathcal{P}_l\left(\mathcal{N}, \Lambda_l\right)$ to be implemented has been determined, the coalitions to be formed by the microgrids are defined. Each of these coalitions $\mathcal{C}_l \in \mathcal{P}_l\left(\mathcal{N}, \Lambda_l\right)$, will make their decisions jointly when solving the optimisation problem presented by Eq. (6).

The cost functional (6a) corresponds to Eq. (4), where the cost ℓ_e^p indicating the economic criterion to be optimised by each microgrid, may be different between microgrids in the same coalition, either because of the characteristics of their components or simply because they pursue distinct management objectives.

$$\min_{\boldsymbol{u}^{\mathcal{C}_l},\, \boldsymbol{z}^{\mathcal{C}_l}} \sum_{i=0}^{N-1} \sum_{p \in \mathcal{C}_l} V_N^p\left(x_{i|k}^{\mathcal{C}_l}, c_{e,k}; \boldsymbol{u}^{\mathcal{C}_l}, \boldsymbol{z}^{\mathcal{C}_l}\right) \tag{6a}$$

$$\text{s.t.: } x_{0|k}^{\mathcal{C}_l} = x_k^{\mathcal{C}_l}, \tag{6b}$$

$$x_{i+1|k}^{\mathcal{C}_l} = A^{\mathcal{C}_l}\, x_{i|k}^{\mathcal{C}_l} + B^{\mathcal{C}_l}\, u_{i|k}^{\mathcal{C}_l}, \tag{6c}$$

$$B_u^{\mathcal{C}_l}\, u_{i|k}^{\mathcal{C}_l} + M^{\mathcal{C}_l}\, z_{i|k}^{\mathcal{C}_l} + E_w^{\mathcal{C}_l}\, w_{i|k}^{\mathcal{C}_l} = 0, \tag{6d}$$

$$x_{i|k}^{\mathcal{C}_l} \in \mathbb{X}^{\mathcal{C}_l},\ u_{i|k}^{\mathcal{C}_l} \in \mathbb{U}^{\mathcal{C}_l},\ z_{i|k}^{\mathcal{C}_l} \in \mathbb{Z}^{\mathcal{C}_l}, \tag{6e}$$

$$x_{N-1|k}^{\mathcal{C}_l} = x_{N|k}^{\mathcal{C}_l}, \tag{6f}$$

$$i \in \mathbb{I}_{0:N-1},\ \forall\, p \in \mathcal{C}_l,\ \forall \mathcal{C}_l \in \mathcal{P}_l\left(\mathcal{N}, \Lambda_l\right).$$

For the control actions $z_k^{\mathcal{C}_l}$, which refer to the links between the microgrids, its decision variable is to determine the values and direction for the energy transfers to occur between the microgrids since the coalitions are defined, the links necessary to form them are all considered to be enabled.

At each time step, the complete sequence for the optimal control actions are calculated $\boldsymbol{u}^{\mathcal{C}_l} = \left\{u_{0|k}^{\mathcal{C}_l^*}, u_{1|k}^{\mathcal{C}_l^*}, ..., u_{N|k}^{\mathcal{C}_l^*}\right\}$, $\boldsymbol{z}^{\mathcal{C}_l} = \left\{z_{0|k}^{\mathcal{C}_l^*}, z_{1|k}^{\mathcal{C}_l^*}, ..., z_{N|k}^{\mathcal{C}_l^*}\right\}$ and according to the receding horizon fashion, only the first actions are applied to the system, while the others are discarded. Therefore, the control laws are: $\kappa\left(x_k^{\mathcal{C}_l}, c_{e,k}\right) = u_{0|k}^{\mathcal{C}_l^*}$ and $\kappa\left(x_k^{\mathcal{C}_l}, c_{e,k}\right) = z_{0|k}^{\mathcal{C}_l^*}$.

IV. Example

A community of 5 microgrids connected at low voltage (mesh topology [5]) with similar architectures, but with different scales, to the one proposed in [1] is considered. Therefore, the variables manipulated u_k^p for each microgrid are the power exchanged with the storage system (P_{bat}) and the grid (P_{grid}). Figure 2 shows a representation of the cluster of microgrids, where the grid is indicated by the power transformer of the distribution station, and each microgrid has access to the grid through this transformer.

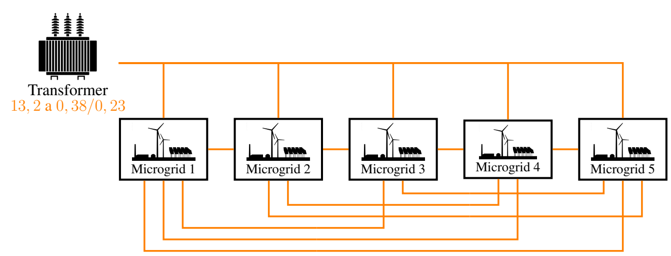

Figure 2. Microgrids interconnected with a mesh topology.

Also, Figure 2 shows that microgrid 1 is the closest to the transformer, while microgrid 5 is the furthest away. The proposed criteria for deciding the coalitions will consider this characteristic of the power losses generated by the resistances in the distribution lines and the connection between microgrids. The distance from the transformer (T) to each microgrid (M) is 1,5 - 2,5 - 4 - 5 and 7 Km, respectively.

The daily power profiles generated by renewable resources and the demand for each microgrid are shown in Figure 3.

The criterion proposed to define the characteristic function of the cooperative n-personal game that determines the cost for a coalition $\boldsymbol{v}\left(\mathcal{C}_l\right)$ according to problem (5), pursues the objectives of (i) maximising the economic gains of each microgrid through the purchase/sale of energy $\left(P^p_{grid(p)}/P^p_{grid(s)}\right)$ in the electricity market, (ii) incentivising energy exchange between microgrids and (ii) minimising the power losses generated by the different energy transfers that can occur between a microgrid and the electricity grid, as well as between microgrids. About these objectives: $\ell_{\mathcal{C}_l}=\sum_{p\in\mathcal{C}_l}\ell_e^p \Rightarrow$

$$\begin{aligned}\ell_e^p=\,&c_{e,k}\,T\left(P^p_{grid(p),k}-P^p_{grid(s),k}-z_k^q+\frac{\left(z_k^q\right)^2 r_{pq}}{v_d^2}\right)\\&+c_{e,k}\,T\left(\frac{{P^p_{grid,k}}^2 r_{pg}}{v_d^2}+\left(1-\eta_t\right)P^p_{grid,k}\right)\end{aligned} \qquad (7)$$

where $c_{e,k}$ denotes the cost of energy in $\frac{\$}{\mathrm{kWh}}$, T is the sampling time, r_{pq} is the electrical resistance for the distribution line connecting the microgrid p with the neighbouring microgrid q expressed in kΩ, v_d denotes the voltage level for the distribution grid in kV, r_{pg} is the electrical resistance in kΩ for the distribution line between the microgrid p and the power transformer intended to supply this grid g and lastly, η_t is the efficiency for this power transformer mentioned.

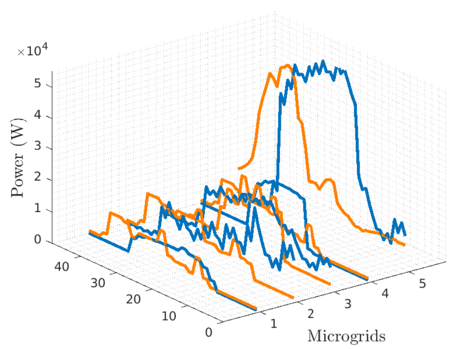

Figure 3. Predictions for no-manipulated variables: Generated (blue) and demanded (orange) power for each microgrid in the community.

For the economic cost ℓ_e^p of the function of Eq. (4), which is used when solving problem (6), the functionals that consider the optimisation of the ageing cycle of the storage system ℓ_{bat}, as proposed in [1], are added.

If the voltage level v_d for the distribution network is 380 Volts, the resistance of the electrical conductors is 1,2 $\frac{\Omega}{\mathrm{Km}}$, the efficiency for the power transformer η_t is 88% and the cost of energy considered to be $c_{e,k}=0,15\,\frac{\$}{\mathrm{kWh}}$; and adopting a simulation time of 48 hours, a control horizon N of 24 hours, a sampling time T of 30 minutes ($T=T_d$), an upper-level time T_t of 6 hours ($b=12$), and applying the proposed decision criterion, the following coalition structures are generated: (i) 00:00 h $\rightarrow \mathcal{P}_0\left(\mathcal{N},\Lambda_0\right)=\{\{2\},\{1,5\},\{3,4\}\}$, (ii) 06:00 h $\rightarrow \mathcal{P}_1\left(\mathcal{N},\Lambda_1\right)=\{\{5\},\{1,3\},\{2,4\}\}$, (iii) 12:00 h $\rightarrow \mathcal{P}_2\left(\mathcal{N},\Lambda_2\right)=\{\{3\},\{1,2\},\{4,5\}\}$ and (iv) 18:00 h $\rightarrow \mathcal{P}_3\left(\mathcal{N},\Lambda_3\right)=\{\{1,2,3,4,5\}\}$, where Figure 4 shows the evolution of these structures for the control horizon.

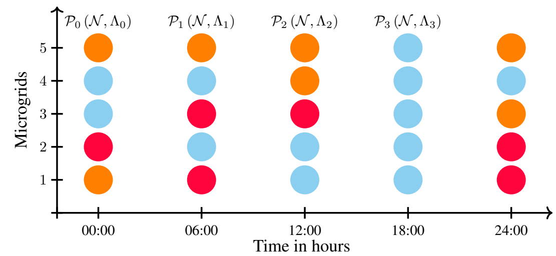

Figure 4. Evolution of coalition structures $\mathcal{P}\left(\mathcal{N},\Lambda\right)$ to implement.

When considering a cooperative game and communication between all agents, it is expected that a grand coalition tends to form, which can be seen in the period when the consumption of microgrids is highest, 18:00 h $\rightarrow \mathcal{P}_3\left(\mathcal{N},\Lambda_3\right)$.

For each coalition formed in a period, the microgrids that belong to it will choose their control actions by solving optimisation problem (6). The simulations were performed in Matlab, and CasADi was used to solve the problems.

V. Conclusions

This paper proposes a coalitional economic model predictive control strategy for controlling a community of microgrids connected to a power grid. This control strategy uses a two-level hierarchical structure. At the upper level, the coalitions to form are decided through a cooperative n-personal game, while at the lower level, the control actions for each coalition are determined. The most notable aspect is the implementation of game theory in conjunction with an optimal control problem with economic criteria to obtain significant improvements.

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