Abstract: Optimal power plant designs are achieved by means of a proposed multiperiod non-linear programming formulation that utilizes the net present value as objective function, while construction, operation and dismantling of the generation facility are accounted for. In addition, optimal operative characteristics are also established for each operative time period, in a way that the system constraints are always satisfied.

Based on the life cycle oriented economic optimal characteristics, a reduced model is proposed as strategy for simplifying the resolution of the rigorous multiperiod model. Trends in the system behavior are identified, enabling the reduction of the multiperiod formulation into a system of non-linear equations plus additional constraints, which allows easily computing accurate estimations of the optimal values of the design variables as well as the time-dependent operative variables.
To: Editor of Applied Thermal Engineering Journal

Subject: Submission of revised version of manuscript “Optimal economic strategy for the multiperiod design and long-term operation of natural gas combined cycle power plants”

Dear Sir,

Please find annexed with this letter the revised version of the manuscript “Optimal economic strategy for the multiperiod design and long-term operation of natural gas combined cycle power plants” for publication, if accepted, in the Applied Thermal Engineering Journal. The authors state that this paper has not been published previously, it is not under consideration for publication elsewhere, and if accepted it will not be published elsewhere in substantially the same form, in English or in any other language, without the written consent of the Publisher.

I look forward to hearing from you.

Yours sincerely

Ezequiel Godoy

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Economic optimal power plants are determined by means of a multiperiod NLP model >
Trends in the system behavior are identified > The original problem is reduced to a system of
equations plus additional constraints > Accurate estimations of the optimal decision variables
are efficiently obtained
Optimal economic strategy for the multiperiod design and long-term operation of natural gas combined cycle power plants

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Abstract

Optimal power plant designs are achieved by means of a proposed multiperiod non-linear programming formulation that utilizes the net present value as objective function, while construction, operation and dismantling of the generation facility are accounted for. In addition, optimal operative characteristics are also established for each operative time period, in a way that the system constraints are always satisfied.

Based on the life cycle oriented economic optimal characteristics, a reduced model is proposed as strategy for simplifying the resolution of the rigorous multiperiod model. Trends in the system behavior are identified, enabling the reduction of the multiperiod formulation into a system of non-linear equations plus additional constraints, which allows easily computing accurate estimations of the optimal values of the design variables as well as the time-dependent operative variables.

Keywords: NGCC power plant, multiperiod optimization, economic model, resolution strategy

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Nomenclature

Acronyms

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
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<tbody>
<tr>
<td>NGCC</td>
<td>natural gas combined cycle</td>
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<tr>
<td>GT</td>
<td>gas turbine</td>
</tr>
<tr>
<td>ST</td>
<td>steam turbine</td>
</tr>
<tr>
<td>HRSG</td>
<td>heat recovery steam generator</td>
</tr>
<tr>
<td>NLP</td>
<td>non-linear programming</td>
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<tr>
<td>MILP</td>
<td>mixed integer linear programming</td>
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Mathematical Symbols

- $f$: objective function
- $x$: set of model variables
- $\hat{x}$: set of model variables - estimated values
- $x^*$: set of model variables - optimal values
- $x_k$: subset of decision variables
- $x_d$: subset of design variables
- $x_{op,oi}$: subset of operative variables at operative period $oi$
- $h$: set of equality constraints
- $g$: set of inequality constraints
- $R_{j,ti}$: Ratios at period $ti$
- $\hat{R}_{j,ti}$: Ratios at period $ti$ - estimated values
- $R_{j,ti}^*$: Ratios at period $ti$ - optimal values
- $\alpha_{j,ti}$: $j$-th adjustment parameter at period $ti$
- $\psi_{j,ti}$: $j$-th functional relationship at period $ti$
- $\gamma_{lj}$: $l$-th parameter for $j$-th Ratio correlation
- $\gamma_{1j}$: parameter for $j$-th Ratio linear correlation
- $\gamma_{2j}$: parameter for $j$-th Ratio linear correlation
Time Periods

- $t_i$: set of time periods
- $p_i$: set of pre-operative time periods
- $o_i$: set of operative time periods
- $l_i$: set of post-operative time periods
- $N_{o_i}$: number of operative periods

Economic Variables

- $NPV$: net present value
- $C_{Inv,A}$: investment cost on transfer area
- $C_{Inv,PT}$: investment cost on process turbines
- $IFC$: investment on fix capital
- $IFC_{p_i}$: investment on fix capital at pre-operative period $p_i$
- $C_{Op,t_i}$: total operative costs at period $t_i$
- $C_{RM,o_i}$: raw materials and utilities costs at operative period $o_i$
- $C_{Mant,o_i}$: maintenance costs at operative period $o_i$
- $C_{OS,o_i}$: operative supplies costs at operative period $o_i$
- $C_{MP,t_i}$: manpower costs at period $t_i$
- $C_{Tax,t_i}$: taxes at period $t_i$
- $C_{GE,o_i}$: general expenses at operative period $o_i$
- $Dep_{o_i}$: depreciations at operative period $o_i$
- $SVFC_{l_i}$: salvage value of fix capital at post-operative period $l_i$
- $Sales_{o_i}$: energy sales at operative period $o_i$
- $NIT_{o_i}$: net income taxes at operative period $o_i$
- $COE_{o_i}$: cost of electricity at operative period $o_i$
1. Introduction

1.1. Economic Decision-Making regarding Energy Systems

Economic optimization becomes critical when designing a new energy system, in order to determine the optimal values of the project financial indicators. Different aspects of this problematic have been addressed in the literature (sensitivity analysis for fuel price [1], relation with thermodynamics [1, 2], different market scenarios [3]), while converging towards a comprehensive framework which may be able to cope with the economic evaluation and optimization as a whole.

A life cycle oriented approach, which makes decisions based on economic indicators that refer to the whole life cycle of the generation system (which usually consists of several phases such as synthesis and design, construction, operation, and eventually disposal [4]), is critical under today’s business conditions due to increased competition and market uncertainties, among others. Moreover, from the economic point of view, decisions made during the early stages of synthesis and design largely determine the economic performance of the plant across its entire life cycle.

A detailed model of the generation system requires NLP formulations, which resolution within a multiperiod time framework turns out to be rather challenging due to their inherent initialization and convergence difficulties. From the state of the art, it is observed that multiperiod design and long-term operation of energy generation systems are achieved through MILP models. In this regards, Iyer and Grossmann [5] formulated a MILP model for the operational planning of utility systems, which aims at determining the optimal schedule that meets the demand at the lowest total cost, while the system gets designed to handle a range of demands because of the uncertain nature of such
parameter. Oliveira and Matos [6] presented an extension of the multiperiod models described by Iyer and Grossmann [5] in order to include the concept of global emissions of the gaseous pollutants, turning the problem of synthesis and operational planning of the utility system into a multi-objective optimization from a superstructure of alternatives, where economic and environmental concerns are considered.

Later, Aguilar et al. [7, 8] addressed the design optimization of flexible utility plants, while simultaneously considering different scenarios and equipment part-load operation in order to deal with energy prices that are driven by the equilibrium among supply and demand.

In all cases, it is observed that the economic optimization of complex energy systems usually turns out to be quite challenging, due to the large number of decision variables and constraints involved and the inherently non-linear nature of the problem (if the model considers a rigorous and detailed description of the plant characteristics).

While trying to cope with this complex task, Godoy et al. [9] introduced a design strategy which allows accurately estimating the economic optimal characteristics of combined cycle power plants, including the associated optimal values of its operative variables, for a wide range of power demands. The authors also took advantage of the characteristics of the families of optimal thermodynamic solutions for power plants [10], and an adequate manipulation of functional relationships among the optimal values of the decision variables. Such strategy is briefly summarized as follows:

- Economic optima are determined for two different power generating facilities, by means of a non-linear programming model where the total annual cost is selected as objective function. This approach allows observing the behavior of the design and operative variables when facing different market conditions as given by the costs ratio (i.e. the relative weight of the costs of investment on transfer area versus the operative costs due to fuel consumption).

- Thermodynamic optima of the combined cycles are determined as their thermal efficiency is maximized, by means of a non-linear programming model, for different values of the specific transfer area (i.e. the ratio between net heat transfer area and generated power).

- Based on the economic optimal solutions, a linear economic optimal relationship between the specific transfer area and the costs ratio is determined. Also, linear functional relationships between the optimal decision variables (including transfer areas of the HRSG sections, power
production of each turbine, fuel consumption, steam mass flow rates, operative pressures and temperatures, etc.) and the specific transfer area are identified.

- A novel reduced model shaped as a system of non-linear equations plus additional constraints is structured when considering the previous items. This strategy allows obtaining accurate estimations of the economic optimal values of the power generating facility design and operative variables, while it enables the reduction of the space of feasible solutions and spares the need of solving the corresponding mathematical optimization problem (which is a difficult task due mainly to initialization and convergence issues).

Godoy et al. [9] concluded that the reduced mathematical formulation facilitates the acquisition of accurate estimations of the power plants design and operative variables while reducing the computational requirements. Moreover, the authors proposed that the obtained data may be applied to efficiently initialize more complex optimization problems, as for example a multiperiod approach for evaluating the performance of the generation system when facing variable market conditions.

1.2. Aim and Outline

In this work, two main tasks are addressed. First, an equations-oriented approach is used to build a rigorous and flexible NLP mathematical model which accounts for the design and operative characteristics of a natural gas combined cycle across its whole useful life. This approach enables the design and operation of such complex engineering system, while allows the identification of its economic optimal distinctive characteristics within a multiperiod time framework. In addition, the detailed economic accounting allows attaining more realistic computations of the project financial performance indicators.

Second, a reduced model of the generation plant is developed based on the discovery of functional relationships among the decision variables which can be exploited as additional constraints in the optimization formulation in order to reduce the space of feasible solutions while drastically reducing the computational effort. This new approach easily allows accurately inferring life cycle oriented economic optimal designs of power plants, while the power demand along the whole time horizon gets fulfilled; and can be considered as an extension of the one presented by the authors at [9].

The resultant system of equations plus additional constraints efficiently provides accurate estimations of the power plant decision variables within a multiperiod time framework and reduces the computation
time, which is mandatory when facing more complex research targets that imply a significant increase of the size of the problem to be solved, as for example:

- Deciding among different process equipment alternatives.
- Analyzing configuration changes in the system within a superstructure of alternatives.
- Considering stochastic uncertainty (by means of Montecarlo or other) in critical parameters.
- Optimizing process operative conditions in real time.
- Accounting for reliability and maintenance within a multiperiod time framework.
- Simplifying initialization strategies of more complex optimization formulations.

2. Economic Multiperiod Optimization Framework

Optimizing the economic performance of the generation system implies a mathematical formulation as given by Eq. (1) through Eq. (3).

$$\max f(\underline{x}, t_i)$$  \hspace{1cm} (1)

$$h(\underline{x}, t_i) = 0$$  \hspace{1cm} (2)

$$g(\underline{x}, t_i) \leq 0$$  \hspace{1cm} (3)

The following considerations are taken into account:

- The multiperiod framework is defined as a set of periods $t_i = \{pi, oi, li\}$ which comprises the main stages of the plant life cycle, including a pre-operative phase $pi$ (when all the construction tasks are carried out), an operative phase $oi$ (when the plant is operated at base load) and a post-operative phase $li$ (when the plant is dismantled).
- $\underline{x} = (\underline{x}_d, \underline{x}_{op,oi})$; $\underline{x}_d$ are the design variables (transfer areas, turbines dimensions, etc.); $\underline{x}_{op,oi}$ are the operative variables for each operative time period (temperatures, pressures, flow rates, etc.).
are constituted by mass and energy balances, properties estimation correlations, and design
equations; while \( g \) correspond to technical and logical constraints.

This mathematical program is implemented in the optimization software GAMS [11] and is solved
by means of the reduced gradient algorithm CONOPT [12]; in an Intel Core i3 3.07 GHz processor
with 2 GB RAM. The implemented mathematical model has approximately 40700 variables and 43800
(equality and inequality) constraints.

2.1. Power Plant Configuration

A 2 GTs + 1 ST multi shaft configuration gas turbine combined cycle power plant is used
as case study and introduced at Figure 1. This power plant consists of two gas turbines with
postcombustion and regeneration, its associated three pressure levels HRSGs, and a steam turbine
with high, intermediate and low pressure stages. This configuration includes innovative features which
enable to obtain high efficiencies (gas to gas recuperation and postcombustion, high gas turbine inlet
temperature, multiple pressure levels and parallel heat exchange sections in the HRSG).

An overview of the features considered in the mathematical model of the NGCC power plant is
introduced at Appendix A, regarding the most significant design features here considered. It is noted
that the mathematical model mainly includes:

- Gas and steam turbines: overall mass and energy balances, and design equations for computing
  the isentropic efficiency and the delivered power are considered. Performance maps provided
  by turbines manufacturers are used to correlate the isentropic efficiency and the flow capacity
  as a function of the compression ratio and rotational speed (see [13, 14], considering [15, 16]),
  for given turbine size and geometry.

- HRSG: overall mass and energy balances are considered for each exchange section, while determining
  the necessary transfer area by computing the overall transfer coefficient and the logarithmic
  temperature difference. Off-design performance is estimated by considering variation of the
  overall transfer coefficients versus the gas flow rate and temperature (as suggested at [16, 17]).

- Auxiliary equipment: mass and energy balances as well as simplified design equations are
  considered for other minor/auxiliary pieces of equipment, although no off-design performance
  calculations are included.
The thermodynamic properties correlations for computing gas and steam properties at the power plant are obtained from the standard literature: [18, 19] for water and steam, [20] for air and combustion gases, [21] for natural gas. Input data for the power plant model has also been fully listed at [9, 10], which have been selected considering the guidelines and suggested values introduced at [3, 16, 22, 23, 24, 25].

Moreover, in order to circumscribe a feasible operative region, and considering the technical limits and recommendations reported at [3, 16, 22, 23, 24, 25], the following inequality constraints are considered:

- Minimum and maximum approach point (5-15 K), to guarantee no water evaporation in the economizers and to avoid thermal shock at evaporator entries, respectively.

- Minimum and maximum pinch point (5-15 K), to secure reasonable practical values of the HRSG heat transfer area.

- Minimum and maximum steam pressure for each operative pressure level at the HRSG (11146-17732 kPa for high pressure, 1520-7093 kPa for intermediate pressure, 304-1013 kPa for low pressure, 101-507 kPa for deaerator), to assure operation within normal parameters.

- Minimum operative pressure of the condenser (5 kPa), fixed by minimum temperature of available cooling water.

- Maximum gas temperature at HRSG inlet (900 K), to prevent materials deterioration.

- Minimum gas pressure at HRSG discharge (101832 Pa), to assure operation within normal parameters.

- Minimum gas temperature at HRSG discharge (360 K), to prevent corrosion due to water condensation.

- Maximum temperature at turbine inlet (1500 K), determined by the materials resistance.

- Minimum temperature difference at superheater exit (30 K), to assure operation within normal parameters.

- Minimum temperature difference at condenser (4 K), to avoid excessive cooling water consumption.
• Minimum temperature difference at regenerator exit (40 K), to assure adequate operative parameters.

• Minimum and maximum steam quality at steam turbine discharge (0.92-0.97), to achieve normal operation of the turbine.

A detailed description of the system along its mathematical model can be seen at [9, 10].

2.2. Power Plant Size and Long-Term Performance

The optimum scale of the plant is an economic decision which results from the resolution of the economic multiperiod optimization formulation, and it is here assumed that the plant size and configuration do not change during its useful life cycle. Moreover, the operative policy regarding the delivered power (i.e. part-load operation versus operation at nominal capacity) is also determined as an optimal output of the optimization program, since the financial tradeoff between variation rates of costs and sales is captured by the objective function. Note that part-load operation gets penalized since performance of turbines and exchangers downgrades for off-design operation (as considered in the previous section).

An upper bound is imposed on the power production along the time horizon, as introduced at Eq. (4), accounting for the long-term expected power demand. Since this work aims at determining economic optimal distinctive characteristics of the power plant at the early stages of the project synthesis and design, such value is assumed as an average annualized one, and does not consider the effect of seasonality or the daily load histogram (which would be the case at a short-term/medium-term planning approach).

\[ \dot{W}_{Net,\alpha} \leq \dot{W}_{0,max} \]  

(4)

Performance of the combined cycle degrades through time as result of the interaction of several factors: equipment fouling, parts replacement, operative policies, etc. Such degradation is here accounted for by means of a diminution of the inherent efficiency of turbines and compressors, following an exponential law according to the guidelines presented at [17].

2.3. Life Cycle Economic Evaluation

The economic performance of the project is evaluated through its net present value (Eq. (5)), which is the summation of net cash flows discounted to present value according to the annual discount
rate desired by the investor (i.e. the summation of discounted cash flows). The net cash flow of the year $t_i$ of the plant life cycle is the difference between the financial inputs and outputs that take place during such period, including sales of electricity, operative costs, investment on fix capital and investment on working capital, salvage value of fix capital, depreciations, and taxes.

$$NPV = \sum \frac{(Sales_{oi} + SVFC_{ti}) - (C_{Op,ti} + IFC_{pi} + NIT_{oi})}{(1 + ADR)^{ti}}$$ (5)

Input data here used in the economic model of the power plant is listed in Table 1. These economic parameters are taken from general and technical literature (for example, [26, 27, 28, 29, 30, 31]); up-to-date electricity prices and fuel costs are obtained from [32]; utilities costs are estimated using correlations introduced at [33]; equipment capital costs are computed considering the formulae and unitary costs reported at [1, 28, 30].

Next, each of the terms considered for computing the net present value of the project is briefly introduced:

- **Investment on Fix Capital**: first, the cost of investment on transfer area and turbines is computed as a function of each process equipment typology (Eqs. (6)-(7)); second, the total investment is computed as the summation of the cost of investment on transfer area and turbines, and is affected by an installation factor (Eq. (8)); lastly, the investment to be expended at each pre-operative time period is determined (Eq. (9)).

$$C_{Inv,A} = C_{u_A} (A_{Net})^a$$ (6)

$$C_{Inv,PT} = C_{u_{PT}} \dot{W}_{PT,D} , \quad PT = GT, ST$$ (7)

$$IFC = F_{Inst} \left( C_{Inv,A} + \sum C_{Inv,PT} \right)$$ (8)

$$IFC_{pi} = F_{Inv,pi} IFC$$ (9)

- **Operative Costs**: the total operative costs are computed as the summation of variable, fix and semi-variable costs (Eq. (10)). Variable costs include raw materials costs (Eq. (11)), maintenance costs (Eq. (12)), and operative supplies (Eq. (13)). Fixed costs include manpower
(Eq. (14)), and fix operative taxes (Eq. (15)). Semi-variable costs include general expenses necessary for supporting administration and normal operation of the facilities (Eq. (16)).

\[ C_{Op,ti} = \sum C_{RM,oi} + C_{Mant,oi} + C_{OS,oi} + C_{MP,ti} + C_{Tax,ti} + C_{GE,oi} \]  
\[ (10) \]

\[ C_{RM,oi} = POT \left( C_F^u (1 + AFCG)^o_i \dot{Q}_{F,oi} + C_W u_i \dot{m}_{W,oi} + C_{CW} u_i \dot{m}_{CW,oi} \right) \]  
\[ (11) \]

\[ C_{Mant,oi} = F_{Mant,oi} IFC \]  
\[ (12) \]

\[ C_{OS,oi} = F_{OS,oi} C_{Mant,oi} \]  
\[ (13) \]

\[ C_{MP,ti} = F_{MP,ti} N_{MP} \]  
\[ (14) \]

\[ C_{Tax,ti} = F_{Tax,ti} IFC \]  
\[ (15) \]

\[ C_{GE,oi} = F_{GE,oi} (C_{Mant,oi} + C_{MP,oi}) \]  
\[ (16) \]

- **Depreciations**: the allocation of the cost of assets is addressed by means of the straight-line method (Eq. (17)); the salvage value is estimated as a fix percentage of the total value of the total investment (Eq. (18)).

\[ Dep_{oi} = \frac{1}{N_{oi}} (1 - FSV) IFC \]  
\[ (17) \]

\[ SVFC_{li} = FSV IFC \]  
\[ (18) \]

- **Sales and Taxes**: revenues are computed from rendering electricity sales (Eq. (19)); net income taxes are estimated in order to deduce them from earnings (Eq. (20)).

\[ Sales_{oi} = POT P_{Elec} (1 + AEPG)^o_i \dot{W}_{Net,oi} \]  
\[ (19) \]

\[ NIT_{oi} = P_{NIT} (Sales_{oi} - (C_{Op,oi} + Dep_{oi})) \]  
\[ (20) \]
2.4. Optimal Designs for Different Case Studies

Case Study I

As first case study, the mathematical problem defined by Eq. (1) through Eq. (3) is solved by maximizing the net present value, for actual market conditions (i.e. actual capital investment and operative costs). Economic optimization when using the net present value as objective function (defined according to Eq. (5)) captures the tradeoff between reducing operative costs and capital investment versus increasing profits due to electricity sales, while satisfying the power demand along the whole time horizon.

Optimal values of the objective function and the economic indicators are listed in Table 2. The costs distribution reflects that the raw materials expenses (in particular, the fuel consumption) comprise the largest portion of the total annualized operative cost, as it is usually found in the industrial practice. Depreciations are computed by means of the straight-line method, over the useful life span of the plant. The project income is calculated solely considering energy sales, while other type of revenues (for example, available power) are not accounted for.

Optimal profiles of the annual cash flows over the power plant life cycle are presented in Fig. 2. During the pre-operative phase, negative cash flows occur because of investment on fix capital as the plant is built. Across the operative phase, increasing positive cash flows are obtained while satisfying the power demand (the cash flow increases about 4.1 % on a yearly basis in the first operative years, although such percentage decreases to 3.7 % in the last ones). Finally, in the last year of the plant technical life cycle, the salvage value of the fixed capital investment originates a positive cash flow as the plant is dismantled.

In addition, optimal values of the design and operative variables associated to the power plant are also listed in Table 2.

The power ratio assumes a constant value across the whole time horizon, which indicates that no structural modifications are considered for the gas and steam turbines. It is also observed that both gas turbines generate about 2/3 of the total expected demand, while the remaining 1/3 is absorbed by the steam turbine. In order to fulfill the 800 MW expected demand, the necessary total gross capacity rises up to 837 MW, in order to cope with the irreversible loses and the auxiliary services requirements.

It can be stated that distribution of heat transfer area gets sequentially accomplished. First, area
is assigned either to the HRSG or the condenser. Then, the area assigned to the HRSG is allocated for conditioning the feed water to the deaerator, to address the heat transfer requirements of the low, intermediate and high pressure operative levels, or for accomplishing reheating of the steam between the high and intermediate pressure stages of the steam turbine.

Respect to the operative variables, the following instances are observed:

- The total fuel consumption presents a cumulative increment of 2.6% from the first operative year up to the last one. In addition, the air flow rate increases 2.8% across the operative phase.
- The compression ratio diminishes from 28.4 at the first operative year to 25.0 at the last one.
- The steam flow rates for the deaerator, and low, intermediate and high pressure levels increase along the operative phase of the power plant life cycle.
- The HRSG operative pressures vary across the operative phase: 13% for low pressure; 8% for intermediate pressure; and 1.5% for high pressure.
- The optimized values of the minimum temperatures differences at each operative pressure level tend towards the lower feasible values across the whole time horizon.

Optimized values of the operative variables are consistent with values reported at [1, 23, 25, 34, 35]. Even though, it is here noted that the operative variables of the NGCC power plant are allowed to adjust their values within wide ranges (as set by the selected minimum and maximum bounds on the technical constraints), which allows exploring a wider space of feasible solutions and enables attaining further improvements of the system performance.

The cost of the generated electricity gets computed according to Eq. (21) as the total operative annualized expenditures per unit of generated energy.

\[
COE_{oi} = \frac{C_{Op,oi}}{W_{Net,oi} POT}
\]  

(21)

Fig. 3 introduces the variation of the electricity cost along the operative phase of the power plant life cycle (note that construction and shut-down periods are not considered, as the cost of electricity cannot be evaluated if the plant is not operative). It is observed that this economic index grows about 1.4% a year, mainly driven by the increment of the fuel cost.
The thermodynamic performance of power plants is commonly evaluated by means of the first-law/thermal efficiency, as defined at Eq. (22).

\[
\eta_{T,oi} = \frac{\dot{W}_{Net,oi}}{\dot{Q}_{F,oi}} 
\]

Then, thermal efficiency profiles associated to the economic optima are presented in Fig. 4 (note that construction and shut-down periods are not considered, as the thermal efficiency cannot be evaluated if the plant is not operative). As consequence of the long-term degradation of the turbines performance, it is observed that the overall efficiency of the system also exhibits a decreasing trend; meanwhile, the specific fuel consumption increases in order to counteract the diminution of the power generation capacity.

Case Study II

As second case study, the mathematical problem defined by Eq. (1) through Eq. (3) is solved by maximizing the net present value, assuming that that annualized growth of the fuel cost \( AFCG \) gets doubled (respect to the value reported at Table 1).

Optimal values of the objective function and the economic indicators, as well as the design and operative variables associated to the power plant, are also listed in Table 2. Optimal profiles of the annual cash flows and electricity costs are presented in Fig. 2 and Fig. 3, respectively.

As expected, it is observed that the largest influence is exerted over the optimal values of the economic indicators. Then, the total operative costs are more than 35% higher, while the net present value is almost 15% lower. Even though, the computational requirements for obtaining the optimal solution at this case study are in the same order of magnitude than at the previous one.

It is also observed that similar conclusions can be drawn respect to the variation of other economic parameters (within reasonable practical interest ranges), as for example, the desired interest rate, the annual growth of the electricity price, among others.

On the other hand, variation of the optimal values of the design and operative variables remains below 2% in all cases throughout the whole time horizon. A priori, it can be concluded that any estimation of the optimal values of the decision variables obtained for the previous case study may still be used in this one, without incurring in significant estimation errors.
3. Reduced Model for the Multiperiod Economic Optimization of Power Plants

A novel strategy is here proposed for easily and accurately acquiring life cycle oriented economic optimal designs of NGCC power plants, following the guidelines previously outlined by Godoy et al. [9]. A flowchart summarizing how this strategy works is presented in Fig. 5, and briefly summarized as follows:

- I. Life cycle oriented economic optima are determined for the NGCC generation system, by means of a non-linear mathematical programming formulation (as discussed at Section 2).

- II. Based on the obtained economic optimal solutions, multiperiod linear functional relationships are identified, as presented at Section 3.1, which describe the time-dependent evolution of practical interest technical Ratios (defined among the decision variables of the power plant).

- III. A system of equations plus additional constraints, as presented at Section 3.2, is build considering the equality and inequality constraints that define the feasible design and operative region, and taking advantage of the multiperiod functional relationships in order to reduce the space of feasible solutions.

Therefore, the NGCC power plant is now represented by the proposed reduced model, which constitutes a straight-forward means to attain accurate estimations of the design characteristics of the power plant while describing the behavior of its operative variables across the whole multiperiod time framework.

3.1. Multiperiod Functional Relationships

Several research have been devoted to the idea of using heuristics derived from the universal thermodynamic properties to obtain profiles for the system operative characteristics respect to the parameters that govern its behavior (see for example [36, 37, 38, 39]). Here, such notions are updated to directly reflect the economic optima attributes, by proposing multiperiod functional relationships.

According to previous results presented by the authors [9], it is possible to define practical Ratios among the decision variables of the power plant, as given by Eq. (23) and listed in Table 3, covering power production distribution, transfer area allocation, expansion ratios, HRSG operative temperatures relations, specific fuel consumption, and air and steam flow rates relations.
These Ratios definitions allow introducing an easy to implement a procedure which aims at providing useful functionalities among the optimal values of the decision variables, as previously presented at [9]. Linked to the optimal solutions of the multiperiod economic optimization formulation, a NLP mathematical problem is presented in order to search for functional relationships between the decision variables of the power plant, as given by Eq. (24) through Eq. (28).

\[ R_{j,ti} = f(x_k, ti) , \quad x_k \subseteq x \]  

\begin{align*}
\min & \sum_j \sum_{ti} \alpha_{j,ti} \\
\ h(\hat{x}, ti) &= 0 \\
\ g(\hat{x}, ti) &\leq 0 \\
\ f(\hat{x}, ti) &\leq f^*(\hat{x}^*, ti) \\
\left[ \psi_{j,ti}(\hat{R}_{j,ti}, \gamma_{lj}) - R_{j,ti}^* \right]^2 &\leq \alpha_{j,ti}
\end{align*}

Here, tolerance parameters are minimized (according to Eq. (24)), which allows finding multiperiod functional relationships (by means of Eq. (28)) that accurately predict the multiperiod economic optimal values of the characteristic Ratios. Within this NLP problem, the sets of equality and inequality constraints (i.e. Eq. (25) and Eq. (26), respectively) remain the same as in the original optimization problem, although they are evaluated for the estimated values of the model variables. Meanwhile, the optimal values of the objective function found when solving the original optimization problem are set as bounds on the values of such function which is now computed for the estimated values of the decision variables (according to Eq. (27)).

This auxiliary formulation is implemented in the optimization software GAMS [11] and is solved by means of the reduced gradient algorithm CONOPT [12]; in an Intel Core i3 3.07 GHz processor with 2 GB RAM. The implemented program has approximately 40700 variables and 45200 (equality and inequality) constraints.

Thus, after achieving optimal values of the power plant characteristics, and if linear functionality
is assumed, the mathematical expression given in Eq. (29) can be used to correlate the Ratios defined among the decision variables as function of time, with a very low computational cost. The values of the adjustment parameters (i.e. $\gamma_{1j}$ and $\gamma_{2j}$), obtained when considering the optima information obtained at Case Study I, are listed in Table 4.

$$\hat{R}_{j,ti} = \gamma_{1j} ti + \gamma_{2j}$$  \hspace{1cm} (29)

Once the adjustment parameters have been determined for all the proposed Ratios, Eq. (29) can be used to compute accurate estimations of the optimal values of the decision variables, as it is exemplified in the next section.

3.2. Resolution Strategy through a System of Equations plus Additional Constraints

A mathematical formulation is here proposed which allows easily and accurately estimating the life cycle oriented economic optima of power plants, including the optimal values of design and operative variables, by simply solving the resultant system of equations plus additional constraints. In addition, it is observed that solving such mathematical formulation is less computationally expensive than the resolution of the original multiperiod economic optimization problem.

Due to their essential, the multiperiod functional relationships (obtained at Section 3.1) summarize the life cycle oriented economic optima characteristics of the NGCC power plant. Thus, as one or more functional relationships are introduced in the original optimization problem, the reduction of the space of feasible solutions is accomplished. Once enough multiperiod functional relationships are introduced in the original optimization problem in order fix to all its degrees of freedom, resolution of this modified mathematical problem becomes equivalent to solving the resultant system of non-linear equations plus additional constraints, which delivers a unique (estimation of the optimal) solution.

Then, a reduced model is proposed, as given at Eq. (30) through Eq. (32). Here, the multiperiod functional relationships are used to reduce the feasible region for the life cycle oriented design and multiperiod operation of the power plant (defined by the equality and inequality constraints of the original optimization formulation).

$$\hat{R}_{j,ti} = \gamma_{1j} ti + \gamma_{2j}$$  \hspace{1cm} (30)

$$h (\mathbf{x}, ti) = 0$$  \hspace{1cm} (31)
The system of equations plus additional constraints is also solved in GAMS [11] by means of the reduced gradient algorithm CONOPT [12]; in an Intel Core i3 3.07 GHz processor with 2 GB RAM. Note that in order to solve this modified problem by means of the software GAMS, a mute variable (i.e. a variable which has no influence on the rest of the model) is used as objective function. The implemented program has approximately 40700 variables and 45200 (equality and inequality) constraints.

Solutions for two different case studies are obtained through the proposed reduced model: Case Study I which is associated to the discussion of the optimal solutions for actual market conditions(i.e. actual capital investment and operative costs); and Case Study II where it is assumed that that annualized growth of the fuel cost AFCG gets doubled (respect to the value reported at Table 1. Values of the economic indicators, the design and operative variables, and the computational performances indexes are listed in Table 5. Moreover, percentage differences between the values of the decision variables obtained through the reduced model and the optimal ones (previously presented in Table 2) are also reported.

The values for the net present value and the internal rate of return rate obtained through the reduced model are the same than the optimal ones obtained when solving the multiperiod formulation. It is also observed that the estimation error for the remaining decision variables remains below 1% for both case studies, and results negligible in most cases.

The obtained optimal values of the economic indicators strongly depend on the assumed values of the economic parameters (listed at Table 1). Even though, it is observed that optimal values of the decision variables can still be obtained when varying the values of such parameters (within reasonable practical interest ranges) without incurring in significant estimation errors.

Resolution of the system of equations plus additional constraints proves to drastically reduce the computational effort when compared to the resolution of the original multiperiod optimization problem. This is appreciated through the significant diminution of the resolution time and the number of iterations, while delivering accurate estimations of the life cycle oriented economic optima of the NGCC power plant.
4. Conclusions

Design and operation of a NGCC power plant are here optimized to meet the expected demand over the technical life cycle of the facility, through a long-term multiperiod NLP optimization model which is developed to cope with this task. This rigorous model accounts for the actual design and operative characteristics of the different pieces of equipment, by means of design equations which consider their performance for off-design operation and its degradation through time.

Afterwards, multiperiod functional relationships among the optimal decision variables are defined in a way they are able to provide accurate estimations of the multiperiod optimal values of the decision variables (such as transfer areas of the HRSG sections, power production of each turbine, fuel consumption, steam mass flow rates, operative pressures and temperatures). These relationships are used to structure an original reduced model, shaped as a system of equations plus additional constraints, which allows easily and accurately estimating the life cycle oriented economic optima of power plants, while drastically reducing the computational requirements when compared to the original rigorous optimization formulation.

This novel approach enables facing new research challenges, as for example real time optimization, stochastic variation of critical parameters, reliability and maintenance considerations within a multiperiod time framework, among others.

Acknowledgements

The authors gratefully acknowledge the financial support of the Agencia Nacional de Promoción Científica y Tecnológica (ANPCyT), the Universidad Tecnológica Nacional (UTN) and the Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET).

A. NGCC Power Plant Modeling Strategy

The most important pieces of equipment considered in the mathematical model of the power plant are the gas and steam turbines and the heat recovery steam generator. A brief description of the equations used for modeling each unit operation is presented, regarding mainly the new characteristics here taken into account.
A.1. Gas Turbine

Expressions of the isentropic efficiency $\eta_{i,AC,oi}$ (Eq. (A.1) and Eq. (A.2)) are used to account for the irreversibilities of the compression processes at the air compressor (AC; see Figure 1), as function of the pressure ratio $PR_{AC,oi}$, the polytropic index of air $kp_{AC}$ and the inherent efficiency of the compressor $\eta_{p,AC,oi}$. These expressions allow accurately computing the gross power consumption of the unit $\dot{W}_{AC,oi}$ (Eq. (A.3)), as function of the air flow rate $\dot{m}_{AC,oi}$ and the actual enthalpy shift $\Delta h_{AC,oi}$ (which is related with the ideal enthalpy shift $\Delta h_{i,AC,oi}$ by means of the isentropic efficiency).

$$\eta_{i,AC,oi} = \left[ (PR_{AC,oi})^{\frac{kp_{AC}-1}{kp_{AC}}} - 1 \right] / \left[ (PR_{AC,oi})^{\frac{kp_{AC}-1}{kp_{AC}}} - 1 \right]$$  \hspace{1cm} (A.1)

$$\eta_{i,AC,oi} = \frac{\Delta h_{i,AC,oi}}{\Delta h_{AC,oi}}$$  \hspace{1cm} (A.2)

$$\dot{W}_{AC,oi} = \dot{m}_{AC,oi} \Delta h_{AC,oi}$$  \hspace{1cm} (A.3)

In addition, expressions of the isentropic efficiency $\eta_{i,j,oi}$ (Eq. (A.4) and Eq. (A.5)) are used to account for the irreversibilities of the expansion processes at each section of the gas turbine (GT1 and GT2; see Figure 1), as function of the pressure ratio $PR_{j,oi}$, the polytropic index of combustion gases $kp_{j}$ and the inherent efficiency of the turbine itself $\eta_{p,j,oi}$. These expressions allow accurately computing the gross power production of each unit $\dot{W}_{j,oi}$ (Eq. (A.6)), as function of the combustion gas flow rate $\dot{m}_{j,oi}$ and the actual enthalpy shift $\Delta h_{j,oi}$ (which is related with the ideal enthalpy shift $\Delta h_{i,j,oi}$ by means of the isentropic efficiency).

$$\eta_{i,j,oi} = \left[ (PR_{j,oi})^{\frac{1-kp_{j}}{kp_{j}}} - 1 \right] / \left[ (PR_{j,oi})^{\frac{1-kp_{j}}{kp_{j}}} - 1 \right] , \hspace{0.5cm} j = GT1, GT2$$  \hspace{1cm} (A.4)

$$\eta_{i,j,oi} = \frac{\Delta h_{j,oi}}{\Delta h_{i,j,oi}} , \hspace{0.5cm} j = GT1, GT2$$  \hspace{1cm} (A.5)

$$\dot{W}_{j,oi} = \dot{m}_{j,oi} \Delta h_{j,oi} , \hspace{0.5cm} j = GT1, GT2$$  \hspace{1cm} (A.6)

Degradation of the gas turbine performance is accounted for by means of a diminution of the nominal inherent efficiency of the turbine itself and the air compressor $\eta_{p,j,D}$, following an exponential law with a given degradation factor $DF$ (Eq. (A.7)) according to the guidelines presented at [17].
η_{p,j,oi} = \eta_{p,j,D} (1 + oi)^{-DF} \quad , \quad j = AC, GT1, GT2 \quad (A.7)

Performance maps provided by turbines manufacturers are used to correlate the isentropic efficiency \( \eta_{i,j,oi} \) (Eq. (A.8)) and the flow capacity \( \dot{q}_{j,oi} \) (Eq. (A.9)) as a function of the pressure ratio \( PR_{j,oi} \) and rotational speed \( N_{j,oi} \) (see [13, 14], considering [15, 16]), for given turbine size and geometry. These correlations consider the nominal values of the isentropic efficiency \( \eta_{i,j,D} \) and the flow capacity \( \dot{q}_{j,D} \) computed for the nominal values of the pressure ratio \( PR_{j,D} \) and rotational speed \( N_{j,D} \).

\[
\eta_{i,j,oi} = f(\eta_{i,j,D}, PR_{j,D}, N_{j,D}, PR_{j,oi}, N_{j,oi}) \quad , \quad j = AC, GT1, GT2 \quad (A.8)
\]

\[
\dot{q}_{j,oi} = f(\dot{q}_{j,D}, PR_{j,D}, N_{j,D}, PR_{j,oi}, N_{j,oi}) \quad , \quad j = AC, GT1, GT2 \quad (A.9)
\]

The net power produced by the gas turbine \( \dot{W}_{GT,Net,oi} \) is computed as the difference of the gross power generated by the turbines itself and the gross power consumed by the air compressor (Eq. (A.10)), affected by the generator electrical efficiency \( \eta_{GT,el} \) and the driver mechanical efficiency \( \eta_{GT,m,oi} \).

\[
\dot{W}_{GT,Net,oi} = \eta_{GT,el} \eta_{GT,m,oi} (\dot{W}_{GT1,oi} + \dot{W}_{GT2,oi} - \dot{W}_{AC,oi}) \quad (A.10)
\]

The operative load of the gas turbine \( L_{GT,oi} \) is computed as the ratio between the effectively delivered power \( \dot{W}_{GT,Net,oi} \) versus its design capacity \( \dot{W}_{GT,D} \) (Eq. (A.11)), while it is restricted to be in the range between minimum and maximum feasible technical values.

\[
L_{GT,oi} = \frac{\dot{W}_{GT,Net,oi}}{\dot{W}_{GT,D}} \quad , \quad 0.5 \leq L_{GT,oi} \leq 1 \quad (A.11)
\]

The mechanical efficiency for electricity generation at the gas turbine \( \eta_{GT,m,oi} \) is assumed to vary as a function of its operative load (Eq. (A.12)).

\[
\eta_{GT,m,oi} = f(L_{GT,oi}) \quad (A.12)
\]

A.2. Steam Turbine

Expressions of the isentropic efficiency \( \eta_{i,j,oi} \) (Eq. (A.13) and Eq. (A.14)) are used to account for the irreversibilities of the expansion processes at each section of the steam turbine (\( ST \ HP \), \( ST \ IP \) and \( ST \ LP \); see Figure 1), as function of the pressure ratio \( PR_{j,oi} \), the polytropic index of steam \( kp_j \)
and the inherent efficiency of the turbine itself $\eta_{p,j,oi}$. These expressions allow accurately computing the gross power production of each unit $\dot{W}_{j,oi}$ (Eq. (A.15)), as function of the steam flow rate $\dot{m}_{j,oi}$ and the actual enthalpy shift $\Delta h_{j,oi}$ (which is related with the ideal enthalpy shift $\Delta h_{i,j,oi}$ by means of the isentropic efficiency).

$$\eta_{i,j,oi} = \left[ (PR_{j,oi})^{\frac{1-kp_{j}}{kp_{j}}} - 1 \right] / \left[ (PR_{j,oi})^{\frac{1-kp_{j}}{kp_{j}}} - 1 \right] \ , \ j = ST \ HP, ST \ IP, ST \ LP$$  \hspace{1cm} (A.13)

$$\eta_{i,j,oi} = \frac{\Delta h_{j,oi}}{\Delta h_{i,j,oi}} \ , \ j = ST \ HP, ST \ IP, ST \ LP$$  \hspace{1cm} (A.14)

$$\dot{W}_{j,oi} = \dot{m}_{j,oi} \Delta h_{j,oi} \ , \ j = ST \ HP, ST \ IP, ST \ LP$$  \hspace{1cm} (A.15)

Degradation of the steam turbine performance is accounted for by means of a diminution of the nominal inherent efficiency of each section $\eta_{p,j,D}$, following an exponential law with a given degradation factor $DF$ (Eq. (A.16)) according to the guidelines presented at [17].

$$\eta_{p,j,oi} = \eta_{p,j,D} (1 + oi)^{-DF} \ , \ j = ST \ HP, ST \ IP, ST \ LP$$  \hspace{1cm} (A.16)

Performance maps provided by turbines manufacturers are used to correlate the isentropic efficiency $\eta_{i,j,oi}$ (Eq. (A.17)) and the flow capacity $\dot{q}_{j,oi}$ (Eq. (A.18)) as a function of the pressure ratio $PR_{j,oi}$ and rotational speed $N_{j,oi}$ (see [13, 14], considering [15, 16]), for given turbine size and geometry. These correlations consider the nominal values of the isentropic efficiency $\eta_{i,j,D}$ and the flow capacity $\dot{q}_{j,D}$ computed for the nominal values of the pressure ratio $PR_{j,D}$ and rotational speed $N_{j,D}$.

$$\eta_{i,j,oi} = f(\eta_{i,j,D}, PR_{j,D}, N_{j,D}, PR_{j,oi}, N_{j,oi}) \ , \ j = ST \ HP, ST \ IP, ST \ LP$$  \hspace{1cm} (A.17)

$$\dot{q}_{j,oi} = f(\dot{q}_{j,D}, PR_{j,D}, N_{j,D}, PR_{j,oi}, N_{j,oi}) \ , \ j = ST \ HP, ST \ IP, ST \ LP$$  \hspace{1cm} (A.18)

The net power produced by the steam turbine $\dot{W}_{ST,Net,oi}$ is computed as the summation of the gross power generated by each section itself (Eq. (A.19)), affected by the generator electrical efficiency $\eta_{ST,el}$ and the driver mechanical efficiency $\eta_{ST,m,oi}$.

$$\dot{W}_{ST,Net,oi} = \eta_{ST,el} \eta_{ST,m,oi} \left( \dot{W}_{ST \ HP,oi} + \dot{W}_{ST \ IP,oi} + \dot{W}_{ST \ LP,oi} \right)$$  \hspace{1cm} (A.19)

The operative load of the steam turbine $L_{ST,oi}$ is computed as the ratio between the effectively
delivered power $\dot{W}_{ST,\text{Net,oi}}$ versus its design capacity $\dot{W}_{ST,D}$ (Eq. (A.20)), while it is restricted to be in the range between minimum and maximum feasible technical values.

$$L_{ST,oi} = \frac{\dot{W}_{ST,\text{Net,oi}}}{\dot{W}_{ST,D}}, \quad 0.5 \leq L_{ST,oi} \leq 1 \tag{A.20}$$

The mechanical efficiency for electricity generation at the steam turbine $\eta_{ST,m,oi}$ is assumed to vary as a function of its operative load (Eq. (A.21)).

$$\eta_{ST,m,oi} = f(L_{ST,oi}) \tag{A.21}$$

### A.3. Heat Recovery Steam Generator

Mass and energy balances (Eq. (A.22)) are considered at every transfer section of the HRSG (including $\text{ECO DEA}$, $\text{ECO LP}$, $\text{ECO1 IP}$, $\text{ECO2 IP}$, $\text{ECO1 HP}$, $\text{ECO2 HP}$, $\text{ECO3 HP}$, $\text{EVA DEA}$, $\text{EVA LP}$, $\text{EVA IP}$, $\text{EVA HP}$, $\text{SH LP}$, $\text{SH1 IP}$, $\text{SH2 IP}$, $\text{SH HP}$, and $\text{RH}$; see Figure 1), considering the exchanged heat $\dot{Q}_{j,oi}$, the flow rate and enthalpy shift of the cold fluid ($\dot{m}_{cf,j,oi}$ and $\Delta h_{cf,j,oi}$, which is here the water/steam side), and the flow rate and enthalpy shift of the hot fluid ($\dot{m}_{hf,j,oi}$ and $\Delta h_{hf,j,oi}$, which is here the combustion gases side). Design equations are used for computing the heat transfer area $A_j$ necessary at each transfer section (Eq. (A.23)), considering the temperatures differences at the cold end $\Delta T_{ce,j,oi}$ and at the hot end $\Delta T_{he,j,oi}$.

$$\dot{Q}_{j,oi} = \dot{m}_{cf,j,oi} \Delta h_{cf,j,oi} = \dot{m}_{hf,j,oi} \Delta h_{hf,j,oi} \tag{A.22}$$

$$\dot{Q}_{j,oi} = U_{j,oi} A_j \frac{\Delta T_{ce,j,oi} - \Delta T_{he,j,oi}}{\ln(\Delta T_{ce,j,oi}/\Delta T_{he,j,oi})} \tag{A.23}$$

Off-design performance is estimated by considering variation of the overall transfer coefficients $U_{j,oi}$ versus the gas flow rate $\dot{m}_{hf,j,oi}$ and the average gas temperature $\bar{T}_{hf,j,oi}$ (Eq. (A.24)), also for every transfer section of the HRSG, as suggested at [16, 17]. These correlations consider the design transfer coefficient $U_{j,D}$ computed for the nominal values of the the gas flow rate $\dot{m}_{hf,j,D}$ and the average gas temperature $\bar{T}_{hf,j,D}$

$$U_{j,oi} = f(U_{j,D}, \dot{m}_{hf,j,D}, \bar{T}_{hf,j,D}, \dot{m}_{hf,j,oi}, \bar{T}_{hf,j,oi}) \tag{A.24}$$

In addition, calculation of the pinch point $PP_{j,oi}$ (Eq. (A.25)) and the approach point $AP_{j,oi}$ (Eq. (A.26)) are included at each evaporator section.
\[ PP_{j,oi} = T_{hf,\text{out},j,oi} - T_{cf,\text{out},j,oi} , \quad j = \text{EVA DEA, EVA LP, EVA IP, EVA HP} \] (A.25)

\[ AP_{j,oi} = T_{cf,\text{out},j,oi} - T_{cf,\text{in},j,oi} , \quad j = \text{EVA DEA, EVA LP, EVA IP, EVA HP} \] (A.26)

### A.4. Overall Balances

The net heat consumption of the power plant \( \dot{Q}_{F,oi} \) is computed as the total energy supplied by the fuel (Eq. (A.27)), by means of the fuel consumption \( \dot{m}_{F,j,oi} \) and its lower heating value \( LHV \).

\[ \dot{Q}_{F,oi} = 2 ( \dot{m}_{F,\text{CC},oi} + \dot{m}_{F,PCC,oi} ) LHV \] (A.27)

The net power production of the power plant \( \dot{W}_{\text{Net},oi} \) is computed as the summation of the net power generated by the gas turbine \( \dot{W}_{\text{GT,Net},oi} \) and steam turbine \( \dot{W}_{\text{ST,Net},oi} \) (Eq. (A.28)). Note that the power plant consists of two gas turbines interconnected to one single steam turbine.

\[ \dot{W}_{\text{Net},oi} = 2 \dot{W}_{\text{GT,Net},oi} + \dot{W}_{\text{ST,Net},oi} \] (A.28)

The net transfer area \( A_{\text{Net}} \) is computed as the summation of the transfer areas of every section at the heat recovery steam generator \( A_j \) and the associated condenser \( A_{CON} \) (Eq. (A.29)). Note that the power plant consists of two HRSGs.

\[ A_{\text{Net}} = 2 \left( \sum A_j + A_{CON} \right) \] (A.29)

### References


[19] IAPWS, Revised supplementary release on saturation properties of ordinary water substance, The International Association for the Properties of Water and Steam, St. Petersburg, Russia, 1992.


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Table 1: Economic Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Units</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>Plant Operative Time</td>
<td>$POT$</td>
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<td>Area Cost</td>
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<td>Installation Factor</td>
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<td>Capital Investment Factor – Second Year</td>
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Table 2: Optimal Values of Decision Variables

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<th>Case Study II</th>
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<td>Net Present Value</td>
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| Design Variables                                         |
|---------------------------------------------------------|--------------|---------------|
| Gas Turbine Gross Design Power                           | MW           | 279.5         | 279.5         |
| Steam Turbine Gross Design Power                         | MW           | 277.6         | 278.3         |
| Power Plant Net Generation Capacity                      | MW           | 800.0         | 800.0         |
| Specific Transfer Area                                   | m²/MW        | 676.4         | 677.0         |

| HRSG Exchange Area Fractions                             |
|---------------------------------------------------------|--------------|---------------|
| Deaerator Section                                        | %            | 20.5          | 20.5          |
| Low Pressure Section                                     | %            | 15.5          | 15.6          |
| Intermediate Pressure Section                            | %            | 19.9          | 19.9          |
| High Pressure Section                                    | %            | 40.7          | 40.7          |
| Reheater Section                                         | %            | 3.3           | 3.3           |

| Operative Variables (average value)                      |
|---------------------------------------------------------|--------------|---------------|
| Fuel Flow                                               | kmol/s       | 0.806         | 0.806         |
| Air Flow                                                | kmol/s       | 19.4          | 19.7          |
| Compression Ratio                                       |              | 26.5          | 26.5          |
| Steam Flow Rate                                         |
| Deaerator Section                                        | kg/s         | 88.6          | 88.6          |
| Low Pressure Section                                     | kg/s         | 7.5           | 7.5           |
| Intermediate Pressure Section                            | kg/s         | 18.3          | 18.4          |
| High Pressure Section                                    | kg/s         | 62.8          | 62.7          |
| Reheater Section                                         | kg/s         | 62.8          | 62.7          |

| HRSG Operative Pressures                                 |
|---------------------------------------------------------|--------------|---------------|
| Low Pressure Section                                     | kPa          | 611           | 607           |
| Intermediate Pressure Section                            | kPa          | 2202          | 2201          |
| High Pressure Section                                    | kPa          | 16101         | 16114         |

| HRSG Minimum Temperature Difference                      |
|---------------------------------------------------------|--------------|---------------|
| Deaerator Section                                        | K            | 5.6           | 5.6           |
| Low Pressure Section                                     | K            | 6.3           | 6.1           |
| Intermediate Pressure Section                            | K            | 5.0           | 5.0           |
| High Pressure Section                                    | K            | 5.1           | 5.1           |

| Computational Performance                                |
|---------------------------------------------------------|--------------|---------------|
| Resolution Time                                         | s            | 173.5         | 204.5         |
| Number of Iterations                                    |              | 1161          | 1362          |
Table 3: Definition of Characteristics Technical Ratios

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>$A_c$</td>
<td>Specific Transfer Area</td>
</tr>
<tr>
<td>$PD$</td>
<td>Power Production Distribution</td>
</tr>
<tr>
<td>$AAR_{k,i}$</td>
<td>Area Allocation Ratio</td>
</tr>
<tr>
<td>$AARO_{k,i}$</td>
<td>Area Allocation Ratio</td>
</tr>
<tr>
<td>$AAROS_{k,i}$</td>
<td>Area Allocation Ratio</td>
</tr>
<tr>
<td>$CR$</td>
<td>Compression Ratio</td>
</tr>
<tr>
<td>$TR_{k,i}$</td>
<td>Operative Temperature Ratio</td>
</tr>
<tr>
<td>$TRH$</td>
<td>Reheater Temperature Ratio</td>
</tr>
<tr>
<td>$MF_s$</td>
<td>Specific Fuel Consumption</td>
</tr>
<tr>
<td>$MA$</td>
<td>Air Flow Rate Relation</td>
</tr>
<tr>
<td>$MS_s$</td>
<td>Steam Flow Rate Relation</td>
</tr>
</tbody>
</table>
Table 4: Values of the Adjustment Parameters for the Linear Relationships

<table>
<thead>
<tr>
<th>Ratio</th>
<th>Parameter $\gamma_{1j}$</th>
<th>Parameter $\gamma_{2j}$</th>
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</thead>
<tbody>
<tr>
<td>$A_s$</td>
<td>$6.764 \times 10^2$</td>
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<tr>
<td>$PD$</td>
<td>$1.023 \times 10^0$</td>
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</tr>
<tr>
<td>$AAR_{s,j}$</td>
<td>$k = HRSG$, $l = Net$</td>
<td>$4.839 \times 10^{-3}$</td>
</tr>
<tr>
<td>$AARO_{s,j}$</td>
<td>$k = DEA$, $l = HRSG$</td>
<td>$2.053 \times 10^{-3}$</td>
</tr>
<tr>
<td></td>
<td>$k = LP$, $l = HRSG$</td>
<td>$1.555 \times 10^{-3}$</td>
</tr>
<tr>
<td></td>
<td>$k = IP$, $l = HRSG$</td>
<td>$1.990 \times 10^{-4}$</td>
</tr>
<tr>
<td></td>
<td>$k = HP$, $l = HRSG$</td>
<td>$4.074 \times 10^{-4}$</td>
</tr>
<tr>
<td></td>
<td>$k = RH$, $l = HRSG$</td>
<td>$3.279 \times 10^{-5}$</td>
</tr>
<tr>
<td>$AAROS_{s,j}$</td>
<td>$k = ECO$, $l = DEA$</td>
<td>$4.853 \times 10^{-4}$</td>
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<tr>
<td></td>
<td>$k = EVA$, $l = DEA$</td>
<td>$5.147 \times 10^{-4}$</td>
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<tr>
<td></td>
<td>$k = ECO$, $l = LP$</td>
<td>$1.216 \times 10^{-3}$</td>
</tr>
<tr>
<td></td>
<td>$k = EVA$, $l = LP$</td>
<td>$7.822 \times 10^{-4}$</td>
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<td>$k = SH$, $l = LP$</td>
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<td>$CR$</td>
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<td>$2.852 \times 10^{1}$</td>
</tr>
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<td>$TR_{x,j}$</td>
<td>$k = DEA$, $l = CON$</td>
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<td></td>
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<td>$3.564 \times 10^{-5}$</td>
</tr>
<tr>
<td></td>
<td>$k = IP$, $l = LP$</td>
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</tr>
<tr>
<td></td>
<td>$k = HP$, $l = IP$</td>
<td>$-8.238 \times 10^{-5}$</td>
</tr>
<tr>
<td>$TRH$</td>
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<td>$-5.768 \times 10^{-4}$</td>
</tr>
<tr>
<td>$MF_{s}$</td>
<td>$k = CC$</td>
<td>$2.406 \times 10^{-3}$</td>
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<tr>
<td></td>
<td>$k = PCC$</td>
<td>$2.605 \times 10^{-4}$</td>
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<tr>
<td>$MA$</td>
<td>$8.602 \times 10^{-3}$</td>
<td>$2.391 \times 10^{1}$</td>
</tr>
<tr>
<td>$MS_{s}$</td>
<td>$k = DEA$</td>
<td>$2.328 \times 10^{-3}$</td>
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<tr>
<td></td>
<td>$k = LP$</td>
<td>$3.320 \times 10^{-3}$</td>
</tr>
<tr>
<td></td>
<td>$k = IP$</td>
<td>$2.171 \times 10^{-2}$</td>
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<tr>
<td></td>
<td>$k = HP$</td>
<td>$-2.270 \times 10^{-2}$</td>
</tr>
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</table>
Table 5: Decision Variables obtained through the Reduced Model

<table>
<thead>
<tr>
<th>Economic Indicators</th>
<th>Case Study I</th>
<th>Difference (%)</th>
<th>Case Study II</th>
<th>Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net Present Value</td>
<td>million US$</td>
<td>1255.75</td>
<td>0.00</td>
<td>1074.25</td>
</tr>
<tr>
<td>Internal Rate of Return</td>
<td>%</td>
<td>17.6</td>
<td>0.00</td>
<td>16.8</td>
</tr>
<tr>
<td>Investment on Fix Capital</td>
<td>million US$</td>
<td>1183.72</td>
<td>0.00</td>
<td>1183.99</td>
</tr>
<tr>
<td>Total Operative Costs (last operative year)</td>
<td>million US$</td>
<td>325.06</td>
<td>0.00</td>
<td>439.91</td>
</tr>
<tr>
<td>Raw Materials</td>
<td>%</td>
<td>70.25</td>
<td>0.00</td>
<td>78.01</td>
</tr>
<tr>
<td>Maintenance</td>
<td>%</td>
<td>7.28</td>
<td>0.00</td>
<td>5.38</td>
</tr>
<tr>
<td>Operative Supplies</td>
<td>%</td>
<td>1.09</td>
<td>0.00</td>
<td>0.81</td>
</tr>
<tr>
<td>Manpower</td>
<td>%</td>
<td>0.39</td>
<td>0.00</td>
<td>0.29</td>
</tr>
<tr>
<td>Fix Operative Taxes</td>
<td>%</td>
<td>16.39</td>
<td>0.00</td>
<td>12.11</td>
</tr>
<tr>
<td>General Expenses</td>
<td>%</td>
<td>4.60</td>
<td>0.00</td>
<td>3.40</td>
</tr>
<tr>
<td>Depreciations (last operative year)</td>
<td>million US$</td>
<td>48.42</td>
<td>0.00</td>
<td>48.44</td>
</tr>
<tr>
<td>Sales (last operative year)</td>
<td>million US$</td>
<td>952.47</td>
<td>0.00</td>
<td>952.47</td>
</tr>
<tr>
<td>Net Income Taxes (last operative year)</td>
<td>million US$</td>
<td>202.64</td>
<td>0.00</td>
<td>162.45</td>
</tr>
</tbody>
</table>

Design Variables

| Gas Turbine Gross Design Power                           | MW           | 279.5          | 0.00          | 279.6          | 0.03           |
| Steam Turbine Gross Design Power                         | MW           | 277.7          | 0.06          | 277.9          | -0.14          |
| Power Plant Net Generation Capacity                      | MW           | 800.0          | 0.00          | 800.1          | 0.02           |
| Specific Transfer Area                                   | m²/MW        | 676.1          | -0.05         | 676.8          | -0.03          |

HRSG Exchange Area Fractions

| Deaerator Section                                        | %            | 20.5           | 0.03          | 20.5           | -0.01          |
| Low Pressure Section                                     | %            | 15.5           | -0.12         | 15.6           | -0.17          |
| Intermediate Pressure Section                            | %            | 19.9           | 0.08          | 20.0           | 0.29           |
| High Pressure Section                                    | %            | 40.7           | -0.01         | 40.7           | -0.07          |
| Reheater Section                                         | %            | 3.3            | 0.02          | 3.3            | -0.02          |

Operative Variables (average value)

| Fuel Flow                                                | kmol/s       | 0.806          | 0.01          | 0.806          | -0.01          |
| Air Flow                                                 | kmol/s       | 19.4           | 0.02          | 19.7           | 0.01           |
| Compression Ratio                                        |              | 26.5           | -0.02         | 26.5           | -0.05          |

Steam Flow Rate

| Deaerator Section                                        | kg/s         | 88.6           | 0.01          | 88.6           | -0.04          |
| Low Pressure Section                                     | kg/s         | 7.5            | -0.02         | 7.5            | -0.16          |
| Intermediate Pressure Section                            | kg/s         | 18.3           | 0.04          | 18.3           | -0.07          |
| High Pressure Section                                    | kg/s         | 62.8           | 0.00          | 62.7           | -0.02          |
| Reheater Section                                         | kg/s         | 62.8           | 0.00          | 62.7           | -0.02          |

HRSG Operative Pressures

| Low Pressure Section                                     | kPa          | 610            | -0.23         | 613            | 0.85           |
| Intermediate Pressure Section                            | kPa          | 2195           | -0.31         | 2212           | 0.47           |
| High Pressure Section                                    | kPa          | 16071          | -0.18         | 16157          | 0.26           |

HRSG Minimum Temperature Difference

| Deaerator Section                                        | K            | 5.6            | -0.01         | 5.6            | 0.27           |
| Low Pressure Section                                     | K            | 6.3            | 0.02          | 6.2            | 0.51           |
| Intermediate Pressure Section                            | K            | 5.0            | 0.00          | 5.0            | 0.00           |
| High Pressure Section                                    | K            | 5.1            | -0.02         | 5.1            | 0.11           |

Computational Performance

| Resolution Time                                         | s            | 1.3            |              | 1.8            |               |
| Number of Iterations                                    |              | 7              |              | 14             |               |
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Figure 1: Flow Diagram for the Power Plant
Figure 2: Economic Optimal Cash Flows

Figure 3: Cost of Electricity
Figure 4: Profiles of Thermal Efficiency at the Economic Optima
I. Life Cycle oriented Multiperiod Economic Optimization Formulation
   - Implies Maximization of Net Present Value
   - Subject to Mass and Energy Balances, Design Equations and Off-design Performance Modeling, considering Technical Operative Restrictions
   - Within a Multiperiod Time Framework which accounts for the time variation of the model parameters and the system performance

Feasible Design and Operative Region
   Defined by the Equality and Inequality Constraints

Optimal Solutions
   Design and Operative Variables for every Time Period

II. Multiperiod Functional Relationships for the Decision Variables
   - Ratios between the Design and Operative Variables are defined
   - Economic Optima are systematized as Ratios Linear Functionalities versus Time

III. Reduced Model - System of Equations plus Additional Constraints
   - The Multiperiod Functional Relationships are used as Additional Constraints
   - While considering the Equality and Inequality Constraints of the Original Optimization Formulation
   - As result, Accurate Estimations of the Optimal Values of the Decision Variables are obtained, while Drastically Reducing the Computational Effort

New Research Opportunities
   - Deciding among different process equipment alternatives.
   - Analyzing configuration changes within a superstructure of alternatives.
     - Considering stochastic uncertainty in critical parameters.
     - Optimizing process operative conditions in real time.
   - Accounting for reliability and maintenance within a multiperiod time framework.
   - Simplifying initialization strategies of more complex optimization formulations.

Figure 5: Flowchart for the Proposed Resolution Strategy