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Arching during the segregation of two-dimensional tapped granular systems: Mixtures versus intruders

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Abstract. We present numerical simulations of binary mixtures of granular disks subjected to tapping. We consider the size segregation process in terms of the arches formed by small and big particles. Although arching has been proposed as one of the chief mechanisms that determines size segregation in non-convecting systems, there is no direct data on arching to support the existing proposals. The pseudo-dynamic approach chosen for this work allows for a straightforward identification of arches in the bulk of the column. We find that, indeed, arch formation and breakage are crucial to the segregation process. Our results show that the presence of large particles induce the formation of more arches than found in mono-sized samples. However, tapping leads to the progressive breakage of big arches where large particles are involved as the segregation process takes place. Interestingly, isolated intruders may or may not rise under tapping depending not only on the size ratio (as it is well known) but also on the degree of ordering of the environment.

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1 Introduction

Segregation is ubiquitous in granular matter [1,2]. Whenever grains of different properties (such as size, shape, density, surface roughness or Young modulus) are put together, handling (shear, pouring, stirring, shaking, etc.) will most likely lead to separation into phases rich in one or the other component. There are several mechanisms proposed that may drive segregation (void filling/percolation, arching, convection, granular temperature gradients, etc.). The importance of each of them depends on the type of perturbation (shear, shaking, pouring, etc.) but also on specific conditions of the perturbation (amplitude and frequency of shaking, shear rate, etc.) and on the granular sample (humidity, relative concentration of the components, etc.). Such vast set of variables makes the whole area of granular segregation a very active research topic still facing a number of challenges on basic understanding.

It is important to distinguish two type of segregation phenomena. On the one hand, asymmetries imposed — generally by the presence of gravity, but also by special confining boundaries or asymmetric perturbations— lead to a ratchet-like mechanisms since the response to an external perturbation depends on whether the perturbation if favored or not by gravity. If particles with different properties respond to a different degree to this ratchet-like mechanism one of the species will migrate relatively faster and segregation will occur; leading two regions in the sample rich in one or the other component. On the other hand, if there are no asymmetries imposed by gravity, confining walls or asymmetric perturbations, grains of one type may have a tendency to come together (as in depletion flocculation for mixtures of colloids of different particle sizes). In this case, the system presents a more patchy structure with islands of one phase rich in one component inside a sea of a phase poor in this component. We will focus on a protocol with intrinsic asymmetry that drives the segregation of mixtures of particles of different sizes: tapping under gravity of a binary mix of disks.

A number of simulations and experiments have explored size segregation, both for a single intruder and for binary mixtures when driving is either a continuous vibration or tapping (see for example [3–10]). This is a classic example of the so called Brazil nut effect, where particles of same material densities, surface properties, etc., that only differ in size, show size segregation with large particles migrating to the top of the column in most situations (exceptions can be found if convection dominates and the orientation of the rolls is tuned by using adequate boundary conditions [10]). The two more general features of this phenomenon are: (i) a single intruder of radius R will rise continuously during agitation if its size relative to the size of the surrounding grains r is larger than a critical value; and (ii) a mixture will segregate for whatever size ratio. For a intruder, the critical size ratio $(R/r)_c$ seems to depend on the angle of repose of the small particles [7, 11]. For spheres (three dimensional systems) the critical size ratio for the intruder is about $(R/r)_c \approx 2.8$ [7], for disks (two-dimensional systems) $3 \lesssim (R/r)_c \lesssim 12.0$ depending on the angle of repose [11]. If the intruder is smaller than $(R/r)_c$, it may raise intermittently, staying long periods at a given height rendering the mean velocity virtually zero. There is still a lack of knowledge regarding the contrasting responses —the existence or not of a critical R/r— between an isolated intruder and a mixture.

A number of proposals to explain the rise of an intruder have been put forward. Setting aside the case where convection rolls drive segregation, the most common explanations for the intruder rise are: (i) void filling [1, 6, 7, 11], and (ii) arching [1, 4, 5]. The void filling approach is based on the idea that small particles below the intruder form a "V-shaped" free surface on which avalanches during vibration will promote the filling of the void below, forcing the large particle to rest at a higher position after each cycle. The arching approach gives a special importance to the formation of an arch between the intruder and neighbor small particles after one cycle that can be broken later to let the small grains in the arch to percolate down pass the intruder. It is clear that simulations such as the one developed by Rosato et al. [6] or Jullien et al. [7] do not induce the formation of arches, yet display segregation and a critical size ratio. The Monte Carlo type simulations from Rosato avoid grains to enter into contact (necessary for arch formation). The sequential deposition scheme by Jullien et al. does not create arches since these structures are cooperative and need two or more grains to come to rest simultaneously [8]. However, real granular materials do form arches and these may be key to the intruder segregation in a real situation as put forward by Durand et al. [4]. Despite the remarkable number of studies done on segregation, there has been not direct measures of arching during segregation.

In this paper, we present a series of simulations of size segregation based on a non-sequential deposition variant of Jullien's simulation, inspired on the Mehta–Barker algorithm [12], that allows for straightforward detection of arches. In this model, as for the basic models from Rosato or Jullien, inertia is neglected, so that reverse Brazil nut effect is not observed. Likewise, convection is not induced in the samples since interaction with the walls do not include a frictional drag in the vibration mechanism. We consider the evolution of arches during tapping of a column of monosized disks with a large intruder as well as a binary mixture of disks. We show that arches play an important role, with large particles forming progressively fewer and smaller arches as segregation occurs in the mixture. We also find that segregation in the mixture for size ratios below the critical size observed in the intruder case seems to be promoted thanks to the enhancement of arching induced by the frustration of ordering due to the presence of nearby intruders.

2 Simulation model

Our simulations are based on an algorithm for inelastic massless hard disks designed by Manna and Khakhar [13]. This is a pseudo-dynamics that consists in small falls and rolls of the grains until they come to rest by contacting other particles or the system boundaries. We use a container formed by a flat base and two flat vertical walls. No periodic boundary conditions are applied.

The deposition algorithm consists in picking a disk and allowing a free fall of length δ if the disk has no supporting contacts, or a roll of arc-length δ over its supporting disk if the disk has one single supporting contact [13, 14]. Disks with two supporting contacts are considered stable and left in their positions. If in the course of a fall of length δ a disk collides with another disk (or the base), the falling disk is put just in contact and this contact is defined as its *first supporting contact*. Likewise, if during a roll a disk collides with another disk (or a wall), the rolling disk is put just in contact. If the *first supporting contact* and the new contact are such that the disk is in a stable position, the second contact is defined as the *second supporting contact*; otherwise, the lowest of the two contacting particle is taken as the *first supporting contact* of the rolling disk and the *second supporting contact* is left undefined. If, during a roll, a particle reaches the same height as the supporting particle, its *first supporting contact* is left undefined (in this way the particle will fall vertically in the next step instead of rolling underneath the first contact). A moving disk can change the stability state of other disks supported by it, therefore, this information is updated after each move. The deposition is over once each particle in the system has both supporting contacts defined or is in contact with the base (particles at the base are supported by a single contact). The coordinates of the centers of the disks and the corresponding labels of the two supporting particles, wall, or base, are saved for analysis.

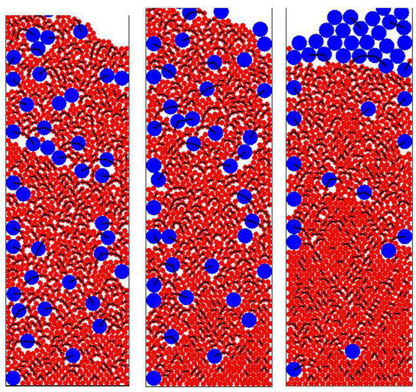

Fig. 1. Snapshots of binary mixture $X = 0.25$ with $R/r = 3$ along the segregation process for $A = 1.1$. (a) After 10 taps. (b) After 100 taps. (c) After 1000 taps. Segments joining particles correspond to the arches identified by our algorithm.

One expects that in the limit $\delta \to 0$ a fairly "realistic" dynamics for fully inelastic non-slipping disk dragged downwards at constant velocity is recovered. This should represent particles deposited in a viscous medium or carried by a conveyor belt. We chose $\delta = 0.0062d$ (with d the diameter of the smaller particle in the system) since we have observed that for smaller values of δ results are indistinguishable from those presented here [14]. In spite of the simplifications of the pseudo-dynamics, it has been shown that results on tapping agree qualitatively with granular dynamics simulations [15].

The simulations are carried out in a rectangular box of width $24.78d$ containing 1500 disks of two different radius r and R. Initially, disks are placed at random (uniformly distributed) in the simulation box (with no overlaps) and deposited using the pseudo-dynamic algorithm. Once all the grains come to rest, the system is expanded in the vertical direction and randomly shaken to simulate a vertical tap. Then, a new deposition cycle begins.

The tapping of the system is simulated by multiplying the vertical coordinate of each particle by a factor A (with $A > 1$). Then, the particles are subjected to several (about 20) Monte Carlo loops where positions are changed by displacing particles a random length Δr uniformly distributed in the range $0 < \Delta r < A - 1$. New configurations that correspond to overlaps are rejected. This disordering phase is crucial to avoid particles falling back again into the same positions. Moreover, the upper limit for Δr (i.e. $A - 1$) is deliberately chosen so that a larger tap promotes larger random changes in the particle positions. The expansion amplitude A ranges from 1.1 to 2.0. For each value of A and each size ratio studied, 5×10^3 taps are applied to the sample.

3 Arches

Arch formation has been identified as a potential cause for segregation in non-convecting systems [1, 4, 5]. However, identification of arches is a rather complex task and no direct test of this proposal has been carried out neither in experiments nor in simulations. Arches can be readily extracted from our pseudo-dynamic simulations since they are defined by construction of the granular sample. In realistic simulations, arches require a careful complex tracking of the history of the formation and breakage of contacts [16, 17]. Experimentally, arches can be identified with great precision when they block an orifice, but require certain approximate criteria if they are in the bulk and the history of contacts is unknown [18, 19]. A preliminary attempt to track contact history and detect arches can be found in Ref. [20].

Arches are sets of mutually stabilizing particles in a static granular sample [21, 22]. In our pseudo-dynamic simulations we first identify all mutually stable particles and then find the arches as chains of particles connected through these mutual stability contacts. Two disks A and B are said mutually stable if A is the left supporting particle of B and B is the right supporting particle of A, or vice versa. Since the pseudo-dynamics rest on defining which disk is a support for another disk during the deposition, this information is available in our simulations.

Details on the properties of arches found in pseudo-dynamic simulations can be found in previous works [14, 15, 23, 24]. In Fig. 1 we have indicated arches by segments that join particles of each arch detected. One interesting feature is that arches are much less preeminent in regions where ordering of the small disks takes place. This is consistent with studies on monosized systems at different tap intensities where an order–disorder transition is accompanied by a sudden drop in number of arches in the ordered phase [15].

4 Segregation for an intruder

We first consider the segregation of a single large particle of radius R surrounded by smaller particles of radius r. It is known that for such system the intruder may raise to the top of the column or not, depending on the size ratio R/r and vibration intensity [1]. We test if this basic phenomenon is well captured by our simulations.

4.1 General observations for the intruder

In Fig. 2(a) we show the height of the intruder (initially placed at the bottom of the simulation box) as a function of the number of taps applied to the system with $A = 1.1$. For these low tap intensities, we observe that the intruder raises to the top of the column continuously only for $R/r > 4.5$. For smaller size ratios the intruder remains at the bottom of the system or rises slightly intermittently but staying at a similar height for long periods. This behavior is consistent with experiments and simulations of others. A critical size ratio that separates the regime in which the intruder raises from the one in which it remains at a low height has been observed in simulations of tapped disks [6, 8, 11] and experiments under continuous vibration [4]. Figure 2(b) shows the mean velocity of the intruder taken as the mean slope in Fig. 2(a). A clear transition is observed at $R/r \approx 4.5$. This value is consistent with those reported by other authors [11]. Notice however that this value will depend on the vibration intensity and has been found to depend on the angle of repose of the small particles [11].

For higher tap intensities the intruder quickly raises to the top even for small R/r (see Fig. 3(a) for an example at $A = 1.3$). It is worth mentioning that for small R/r the intruder may sink partially after reaching the top. This is not due to convection rolls, which are not present in these simulations. Figure 3(b) shows the trajectory for one of these partial sinking intruders.

4.2 Arching for the intruder

In Fig. 4(a) we show the mean size of the arch formed by the intruder (in number of particles) as a function of the tap number. The size of the arch is averaged over a dozen configurations taken every ten taps at chosen intervals along the process. In Fig. 4(b) we also plot the percentage of configurations in which the intruder does not

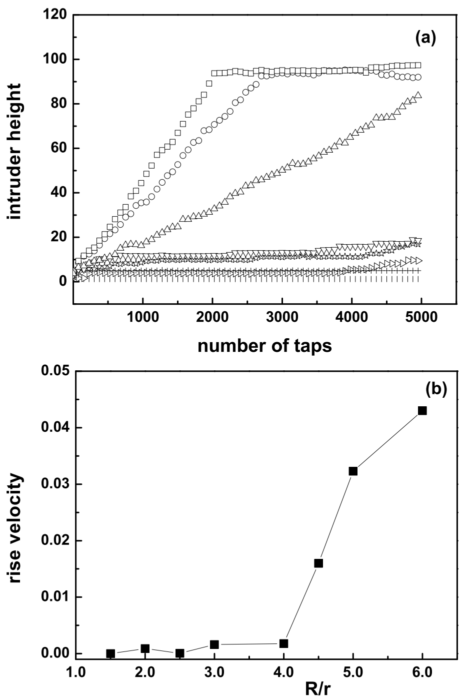

Fig. 2. (a) Intruder height as a function of tap number for various $A = 1.1$ and $R/r = 6.0$ (squares), 5.0 (circles), 4.5 (up triangles), 4.0 (down triangles), 3.0 (stars), 2.5 (crosses), 2.0 (righ triangles), 1.5 (bars). (b) Intruder mean velocity as a function size ratio R/r for $A = 1.1$.

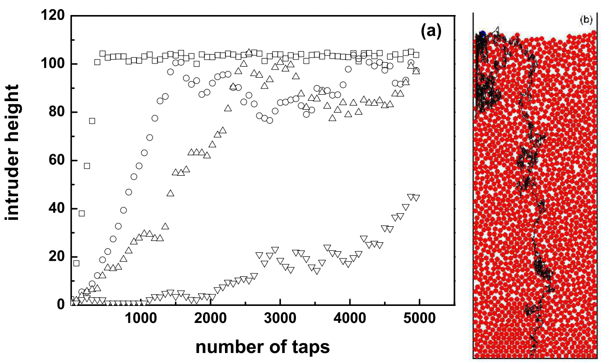

Fig. 3. (a) Intruder height as a function of the number of taps for $A = 1.3$ and various $R/r = 3.0$ (squares), 1.5 (circles), 1.25 (up triangles), 1.1 (down triangles). (b) Trajectory of an intruder for $A = 1.3$ and $R/r = 1.25$.

form any arch. Besides the fluctuations, there are clear trends. For $R/r = 3$ and $A = 1.1$ the intruder does not rise to the top and we can see that the arches it forms comprise between two and three disks all along the process. For a larger intruder, $R/r = 5$, and the same low tap intensity $A = 1.1$, where segregation does occur, the arches the large particle forms are in average higher.

Figure 4(b) provides information on how often the intruder is part of an arch. For the rising intruder of $R/r = 5.0$, the intruder forms an arch essentially after every tap. Interestingly, the smaller intruder ($R/r = 3$) does not form arches in about 40% of the configurations roughly after tap 1000. It is in this part of the process where the intruder stays about the same height (see Fig. 2(a)). Before tap 1000 the small intruder did show a mild rise in coincidence with a low percentage of configurations in which it does not form arches. This suggests that formation of arches is key to the intruder rise.

Duran et al. put forward the idea that to be able to rise upon a tap, an intruder needs to form an arch with a small particle that would fit in the cone left by an ordered environment of small mono-sized grains [4]. This allows for the prediction of a critical size ratio. However, the same prediction can be made without invoking arching [11]. According to our results, intruders do form arches with the small particles in the process of migrating upwards. Although this fact may not be crucial to the prediction of a critical size ratio, we expect that a detailed model that aims at capturing most of the phenomenology behind the Brazil nut effect has to take this fact into account. In particular, the intermittent mild rise observed for $R/r < (R/r)_c$ may be directly related to the arch formation and breakage.

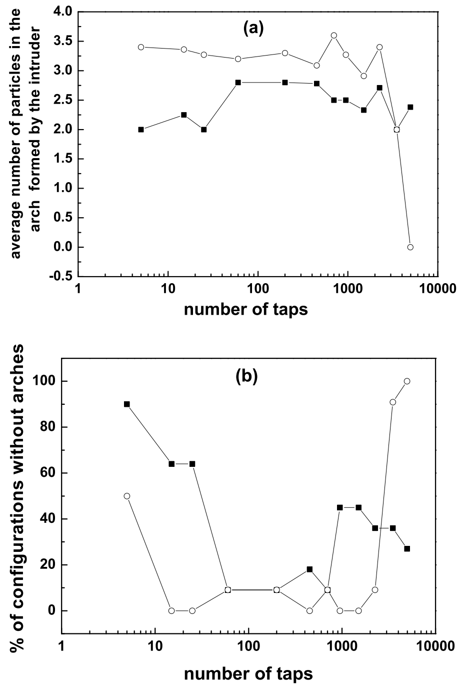

Fig. 4. (a) Average size (in number of particles) of the arch formed by the intruder at $A = 1.1$ for $R/r = 3.0$ (filled squares) and 5.0 (empty circles). (b) Percentage of configurations in which the intruder does not form arches.

5 Segregation for a binary mixture

We consider binary mixtures of disks with $R/r = 3.0$. $N_l = 40$ disks of radius R and $N_s = 1460$ disks of radius r are placed initially at random in the simulation box and deposited via the pseudo-dynamic algorithm. We define the relative concentration X as the ratio of area occupied by large and small disks (i.e., $X = (N_l/N_s)(R/r)^2 = 0.25$). After each tap, we analyze the resulting configuration. In order to quantify the segregation process we use two different indices that will be measured as a function of the number of taps. The first index is

$$I_1 = 2\frac{H_s - H_l}{H_s + H_l}. \quad (1)$$

This is the same index defined by Cimarra et al. [3], where H_s and H_l are the mean height for small and large particles, respectively. In the case of a system well mixed in the vertical direction $I_1 = 0$. When large particles tend to segregate to the top of the column during tapping I_1 becomes negative. Its minimum value will depend on the particular parameters of the mixture, i.e., size ratio and relative concentration of disks.

We define a second index I_2 to show the evolution of the contacts between particles of different sizes. We count the number of contacts N_{sl} between disks of different radius and divide this number by the number of particles N, i.e., $I_2 = N_{sl}/N$. As the segregation takes place, I_2 decreases. After full segregation I_2 fluctuates around the number of contacts at the interface between the two phases (the upper phase of large particles and the bottom phase of small particles).

5.1 General observations for mixtures

In Fig. 5 we show a log-linear plot of the results for I_1 and I_2 as functions of the number of taps at different tap amplitudes for a $X = 0.25$ binary mixture with $R/r = 3.0$.

We find that the number of taps needed to achieve a full segregation of the column decreases with A. For $A = 1.1$ the system does not achieve a full segregation even after 5000 taps. Notice that for this size ratio $R/r = 3.0$ the simulations with a single intruder did not yield segregation for $A = 1.1$. Other authors have pointed out this contrast in the behavior of mixtures and intruders; whereas intruders have a minimum size ratio that leads to segregation, mixtures segregate even for smaller R/r [1]. A definitive answer as to what drives this contrasting behaviors is still lacking. In section 6 we will provide a clue by looking into systems with different initial conditions.

All over the segregation process, I_2 presents larger fluctuations than I_1. While I_1 decays slightly initially, the contacts between disks of different sizes —characterized by I_2— do not change significantly during the first 50 tap, even at large tap intensities. This is due to the fact that, although large disks move upwards as soon as tapping begins —making I_1 decrease— they all remain surrounded by small disks during the initial stages of the segregation process. I_1 captures any large scale vertical segregation, while I_2 only features local segregation (such as cluster formation or domain growth). The results shown in Fig. 5 indicate that segregation by clustering is not present in our simulations; only the formation of a domain of large disks at the top of the system drives the decrease of I_2. The behavior of I_1 and I_2 for a greater concentration of large disks is rather similar to the one shown in Fig. 5.

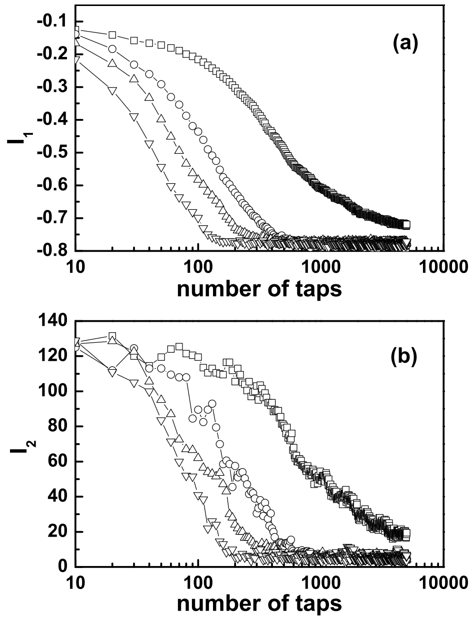

Fig. 5. (a) I_1 as a function of tap number for a $X = 0.25$ binary mixture of size ratio $R/r = 3.0$ for $A = 1.1$ (squares), 1.3 (circles), 1.5 (up triangles), 2.0 (down triangles). (b) Same as (a) but I_2 is reported.

5.2 Arching for mixtures

Figure 6(a) displays the fraction of large particles not forming arches. We can observe that most large particles form arches (mostly in cooperation with small particles but possibly also with other large particles). As tapping proceeds, large particles are progressively less involved in arches. Eventually, after full segregation, both phases present the mean number of arches observed in mono-sized systems. Likewise, the number of particles involved in arches that contain at least one large particle drops quickly during segregation (see Fig. 6(b)) indicating that there are less arches or the arches containing large particles are formed by fewer disks.

It is important to emphasize that the decrease in the number of large particles forming arches is not due simply to the large particles that reach the top of the column. Well before the first large particles emerge from the packing —i.e., while I_2 remains constant— the fraction of large particles involved in arches significantly decreases. This implies that tapping progressively breaks arches that contain large particles. This can be better observed when

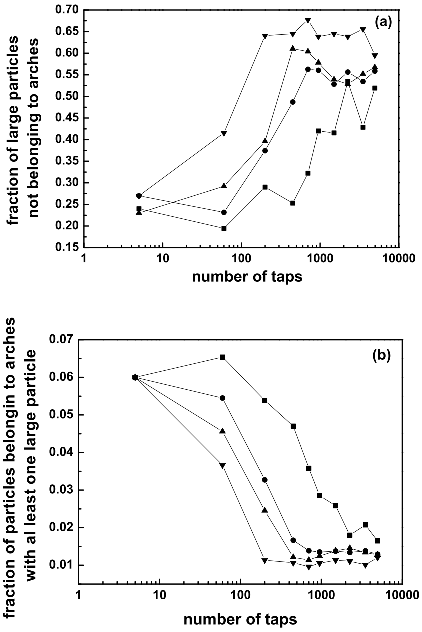

Fig. 6. (a) Fraction of large particles not forming arches as a function of the number of taps for binary mixture with $R/r = 3$ and $X = 0.25$. (b) Fraction of all particles that belong to an arch that includes at least one large particle. Results for different tap intensities $A = 1.1$ (squares), 1.3 (circles), 1.5 (up triangles), 2.0 (down triangles) are shown.

arches in a monosized system are compared against those in binary mixtures.

Figure 7 presents the fraction of particles involved in arches (regardless of the size of the disks) for monosized and mixed systems at different tap amplitudes. When the samples are initially prepared, monosized systems have fewer particles forming arches in comparison with the mixtures. After full segregation, for given A, one expects that the two pure phases behave as mono-sized samples and the fraction of particles in arches should equals that of a mono-sized system. This is indeed observed in all cases but the one for lower tap intensity ($A = 1.1$) for which segregation progresses very slowly. Figure 7 clearly shows that, for $A > 1.1$, the number of particles in arches remains constant after a few dozen taps for monosized systems, but decreases continuously over hundreds of taps for the mixture. For $A = 1.1$ a peculiar phenomenon occurs: for the monosized system the number of particles forming arches initially grows and then falls rapidly to finally remain constant, whereas the mixture presents a constant number of particles in arches initially and then drops sharply after about 1000 taps. The fall in number of particles in arches for monosized systems is related with the growth of ordered regions where arches are less preeminent [14]. The delay observed in the mixed system to present this sharp drop is due to the frustration of order induced by the presence of the large particles. Only after a large proportion of the system has been fully depleted of large grains (the bottom part), the small grains can grow a crystalline structure and so quickly reduce the overall number of arches.

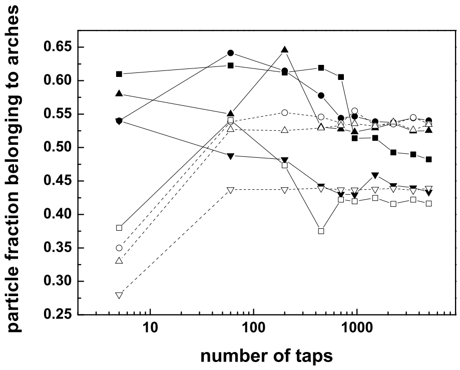

Fig. 7. Fraction of all particles forming arches (regardless of particle sizes) for a mono-sized (open symbols) and a binary sample with $R/r = 3$ and $X = 0.25$ (filled symbols) for different tap intensities: A =1.1 (squares), 1.3 (circles), 1.5 (up triangles) and 2.0 (down triangles).

Figure 8(a) presents the size distribution n_s of arches defined as the fraction of arches composed by s particles. We have averaged this distribution over a dozen consecutive taps at three different stages of the segregation process (indicated in the figure by the number of taps applied before the average is taken). We can see that the distribution of sizes is rather stable for the mixture during tapping with a very small decrease in the number of arches consisting of three or more disks. However, if we only consider arches that contain at least one large particle (see Fig. 8(b)), the drop in the fraction of arches containing three or more particles is dramatic. Tapping favors breaking of large arches containing large particles, and formation of small two-particle arches (one large and one small particle). That large particles are part of larger arches at the initial stages — when most of their neighbors are of small radius — compared with the arches formed after segregation can be observed in the snapshots of Fig. 9.

In summary, during segregation, the binary mixture tends to reduce the number and the size of the arches formed by large particles. Interestingly, this is in contrast with the observations for an intruder where rising is associated with the constant formation of arches.

6 An intruder in a disordered environment

In view of the different segregation response that intruders have in comparison to mixtures, we have considered the effect that a second intruder (and also a different environment) has on the rise of an intruder. In Fig. 10(a) we show the trajectory of a single intruder of "subcritical" size ratio ($R/r = 3.0$) initially placed at mid height into the granular column tapped at low amplitude ($A = 1.1$). In comparison with the same intruder placed at the very

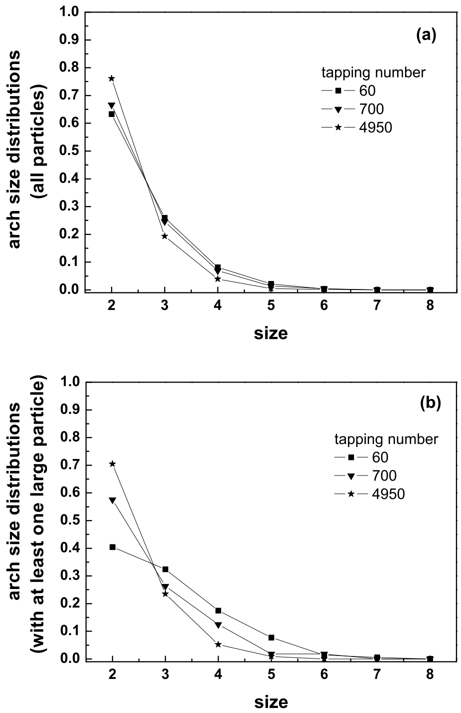

Fig. 8. (a) Size distribution of arches (regardless of the size of the particles). (b) Size distribution of arches containing at least one large particle. Results for $X = 0.25$ binary mixture with $R/r = 3.0$ and $A = 1.1$.

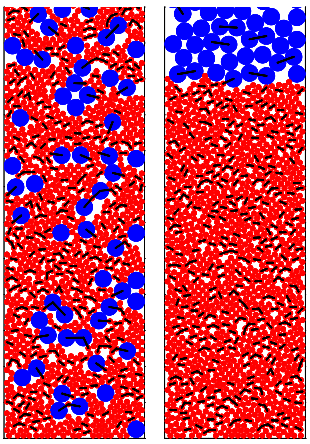

Fig. 9. Snapshots for $A = 1.2$, $R/r = 3.0$ and $X = 0.25$. (a) After 10 taps. (b) After 5000 taps. Arches are indicated by joining segments.

bottom of the sample, this new initial condition leads to a much faster rise and full segregation. We have observed that the development of order in the monodisperse regions of small particles tends to start at the base and grows upwards. An intruder placed at the base may soon be trapped in a somewhat ordered surrounding, less prone to arch formation, that would require strong taps to lead to a net rise of the large particle. In contrast, placing the intruder at a higher initial position, ensures that it will remain surrounded by a disordered environment having a greater probability for arching. This leads to a completely different dynamics with a high rise velocity that drives the large particle up to the free surface well before any ordering can take place around the intruder.

The observation above suggests that the role of other large particles in a mixture is to frustrate ordering of the small grains and so enhance segregation even for small size ratios well below the apparent critical $(R/r)_c$. In Figs. 10(b)-(d) we show the trajectory of two intruders ($R/r = 3.0$ and $A = 1.1$) placed at different initial positions. The rise of any of the intruders strongly depends on the relative positions. In particular, having a intruder placed a few small particle diameters below the other (see Fig. 10(c)) seems to be very effective at promoting the rise of the upper intruder. The bottom large particle will nevertheless remain trapped.

7 Conclusions

Although arching is often claimed to have an important role in many phenomena in granular materials, it is seldom found that these are directly measured and correlated with standard observables. We have studied the arches that a system of disks of different sizes form during the segregation process driven by tapping.

As most previous studies on segregation of a intruder, we found a critical size ratio (which depends on the tap amplitude) below which the large grain does not rise. However, if the intruder is set in an environment that remains disordered for enough time (slight taps always induce ordering of monosized regions), it will rise even for subcritical size ratios. This is key to the contrasting behavior of mixtures, which display segregation even for $R/r < (R/r)_c$ at low tap intensities.

If the intruder segregates, it forms arches of similar sizes all along the rise essentially after every tap. If it does not segregate, the size of the arches it forms is somewhat smaller and a large portion of the configurations correspond to the intruder not forming an arch.

In mixtures, the large particles are part of fewer and smaller arches as tapping proceeds. For the latest stages of the segregation, and particularly for strong tapping, this is clearly due to the fact that a number of these large grains are already segregated at the top part of the sample. However, for low tap intensities where segregation is slow, the overall number of grains involved in arches remains constant at the same time that the large particles form in average smaller and fewer arches.

Our results suggest that a basic model for intruder segregation should consider arching as done by Duran et al. [4]. However, these authors took an ordered structure of small particles as a starting point for their calculations. Taking into account that a disordered environment will alter the intruder ability to form arches may lead to the prediction that a critical size ratio is only valid in a narrow range of situations of interest where ordering is prominent.

8 References

References

1. Kudrolli, Rep. Prog. Phys. **67**, 209 (2004).
2. K. E. Daniels and M. Schröter, New J. Phys. **15**, 035017 (2013).
3. M. Pica Ciamarra, M. D. De Vizia, A. Fierro, M. Tarzia, A. Coniglio, and M. Nicodemi, Phys. Rev. Lett. **96**, 058001 (2006).
4. J. Duran, J. Rajchenbach, and E. Clément, Phys. Rev. Lett. **70**, 2431 (1993).
5. J. Duran, T. Mazozi, E. Clément, and J. Rajchenbach, Phys. Rev. E **50**, 5138 (1994).
6. A. Rosato, K. J. Strandburg, F. Prinz, R. H. Swendsen, Phys. Rev. Lett. textbf58, 1038 (1987).
7. R. Jullien, P. Meakin and A. Pavlovitch, Phys. Rev. Lett. **69**, 640 (1992).
8. G. C. Barker and A. Mehta, Europhys. Lett. **29**, 61 (1995).
9. G. C. Barker, Anita Mehta, and M. J. Grimson, Phys. Rev. Lett. **70**, 2194 (1993).
10. J. B. Knight, H. M. Jaeger, and S. R. Nagel, Phys. Rev. Lett. **70**, 3728 (1993).
11. R. Jullien, P. Meakin and A. Pavlovitch, Europhys. Lett. **22** 523 (1993).
12. A. Mehta and G. C. Barker, Phys. Rev. Lett. **67** 394 (1991).
13. S. S. Manna and H.J. Herrmann, Eur. Phys. J. E **1**, 341 (2000).
14. L. A. Pugnaloni, M. G. Valluzi and L. G. Valluzzi, Phys. Rev. E **73**, 051302 (2006).
15. L. A. Pugnaloni, M. Mizrahi, C. M. Carlevaro and F. Vericat, Phys. Rev. E **78**, 051305 (2008).
16. R. Arévalo, D. Maza and L. A. Pugnaloni, Phys. Rev. E **74**, 021303 (2006).
17. C. M. Carlevaro and L. A. Pugnaloni, Eur. Phys. J. E **35**, 44 (2012).
18. Y. X. Cao, B. Chakrabortty, G. C. Barker, A. Mehta and Y. J. Wang, Europhys. Lett. **102**, 24004 (2013).
19. M. C. Jenkins, M. D. Haw, G. C. Barker, W. C. K. Poon and S. U. Egelhaaf, Phys. Rev. Lett. **107**, 038302 (2011).
20. M. A. Aguirre, L. A. Pugnaloni, T. Divoux and J. G. Grande, In: Powders and Grains 2009, Proceedings of the 6th International Conference on Micromechanics of Granular Media, Eds. M. Nakagawa and S. Luding, pp. 227 (2009).
21. L. A. Pugnaloni, G. C. Barker and A. Mehta, Adv. Complex Syst. **4**, 289 (2001).
22. L. A. Pugnaloni and G. C. Barker, Physica A **337**, 428 (2004).
23. R. O. Uñac, A. M. Vidales and L. A. Pugnaloni, Granular Matter **11**, 371 (2009).
24. A. Garcimartín, I. Zuriguel, L. A. Pugnaloni and A. Janda, Phys.Rev. E **82**, 031306 (2010).

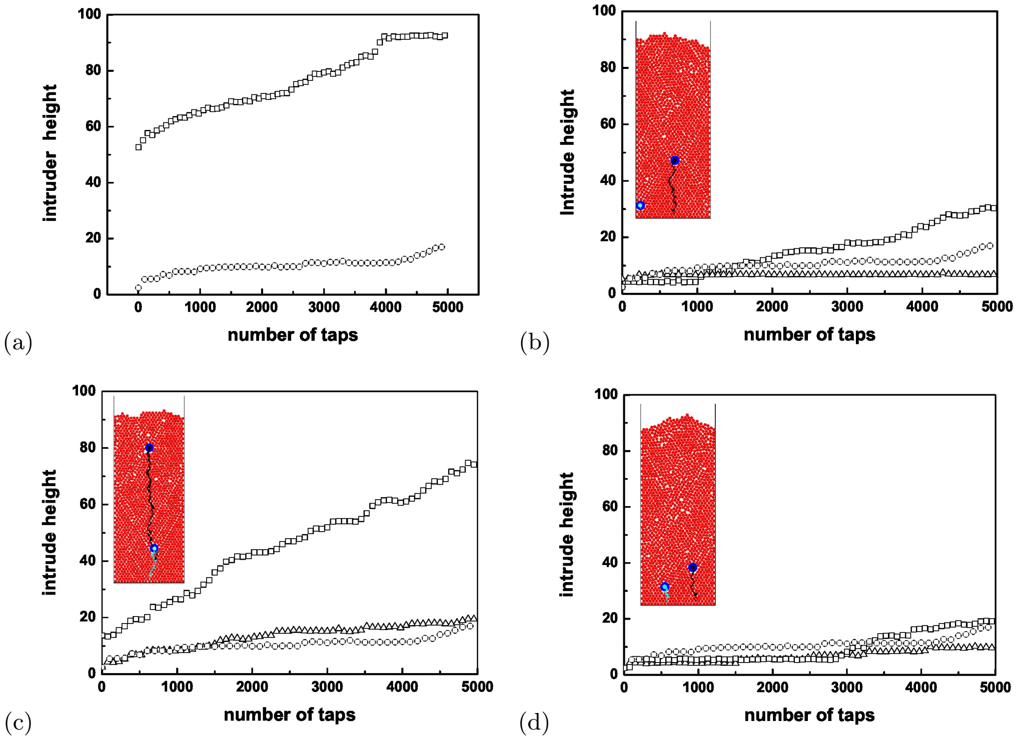

Fig. 10. (a) Rise (z-position vs. number of taps) of an intruder ($R/r = 3.0$) subjected to $A = 1.1$ initially placed in the middle of the column (squares). (b)-(d) Rise of two intruders (squares and triangles) with $R/r = 3.0$ and $A = 1.1$ for different initial positions as indicated in the insets. In all plots the trajectory for an intruder initially placed at the bottom of the system (as studied in section 4) is shown with open circles.