An MILP model for planning of batch plants operating in a campaign-mode

Yanina Fumero
Instituto de Desarrollo y Diseño – CONICET – UTN
yfumero@santafe-conicet.gov.ar

Gabriela Corsano
Instituto de Desarrollo y Diseño – CONICET – UTN
gcorsano@ santafe-conicet.gov.ar

Jorge M. Montagna
Instituto de Desarrollo y Diseño – CONICET – UTN
mmontagna@ santafe-conicet.gov.ar

Abstract

A mixed integer linear programming for the detailed production planning of multiproduct batch plants is proposed in this work. New timing decisions are incorporated to the model taking into account that an operation mode based in campaigns is adopted. For plants operating in a regular fashion along a time horizon, this operation mode assures a more efficient production management. In addition, sequence-dependent changeover times and different unit sizes for parallel units in each stage are considered. Given the plant configuration and unit sizes, the total amount of each product to be produced and the product recipes, the proposed model determines the number of batches that compose the production campaign and their sizes, the assignment and sequencing of batches in each unit, and the timing of batches in each unit in order to minimize the campaign cycle time. The proposed model provides a useful tool for solving the optimal campaign planning of installed facilities.

Keywords: multiproduct batch plants; production campaign; planning; scheduling; MILP model.

1 Introduction

Multiproduct batch plants are characterized by their flexibility to manufacture multiple products using the same equipment. These plants consist of a collection of processing units where batches of the various products are processed by executing a set of operations. These operations can be characterized by a processing time and they do not involve both simultaneous feed and removal of products from the unit during this processing time. Units that perform the same operation are grouped in a production stage, and they can operate in parallel mode (in phase or out of phase). In multiproduct batch plants, every product follows the same sequence through all the processing stages (Voudouris and Grossmann, 1992). Assuming a given plant, i.e., its configuration and the unit sizes are known, different production problems can be posed depending of the contemplated scenario. In particular, when products demands can be accurately forecasted during a relatively long time horizon due to a stable context, more efficient management and control of the production resources can be attained if the plant is operated in a periodic or cyclic way, i.e. in a campaign-mode. In this case, the campaign consists of several batches of different products that are going to be manufactured and the same pattern is repeated at a constant frequency over a time horizon. This campaign-based operation mode has several advantages, for example, more standardized production during certain periods of time, easier and profitable operations decisions, more efficient operation control, and adequate inventory levels without generating excessive costs and minimizing the possibility of stock-outs.

Under this context, a cyclic scheduling problem must be addressed. This type of scheduling is used for products manufacturing with relatively constant demand during a planning horizon, which lead to a more regular production mode and it is more appropriate for a make-to-stock production policy. From the computational point of view, the cyclic scheduling allows reducing the size of the overall scheduling problem, which is often intractable. On the other hand, from the modeling point of view, one of the main differences between cyclic scheduling based on mixed product campaigns (MPCs) and short-term scheduling is the adopted objective function. While the most of approaches for short-term scheduling dealt with makespan minimization, tardiness or earliness, the most appropriate performance measure for the scheduling problem using MPCs of cyclic repetition is the minimization of the campaign cycle time (Fumero et al., 2012). Taking in mind that in a planning context the campaign will be repeated over the time horizon, consecutive campaigns have to be overlapped in order to reduce idle times between them as much as possible.

According to Maravelias (2012), the scheduling problem in the context of batch process involves the following decisions: (i) selection and sizing of batches to be carried out; (ii) assignment of batches to process units; (iii) sequencing of batches on units; and (iv) timing of batches. Taking into account the combinatorial nature of the problem, most of the existing approaches in the process systems engineering literature consider a special case of the problem, where the number and size of batches is fixed, i.e. the lot-sizing problem is solved first and then obtained batches are used as inputs in the scheduling model.

The scheduling problem using MPCs was scarcely addressed in the literature. Besides the paper of Fumero et al. (2012), Birewar and Grossmann (1989) developed slot-based formulations MILP for scheduling of multiproduct batch plants using production campaigns, considering different transfer policies (unlimited intermediate storage, UIS, and zero wait, ZW) and where the number and size of batches are data problem. They determined the optimal campaign cycle time, for simple plants including only one unit per processing stage. In Fumero et al. (2011) two MILP models for the simultaneous design and scheduling of a multi-stage batch plant are proposed. The parallel units are considered identical and no changeover times are taken into account. The rest of the papers that mention the use of campaigns, do not refer to the determination of batches and its cyclic sequencing, as it is managed in this work.

In this work, the detailed planning problem of multistage batch plants with an operation based on MPCs is addressed using a MILP model. It is assumed that the plant manager must produce known demands using a cyclic campaign during a time horizon. Nonidentical parallel units, ZW transfer policy and sequence-dependent changeover times are considered. Given the plant configuration and unit sizes, the total amount of each product to be produced in the campaign and the product recipes, the approach determines the number of batches that compose the production campaign and their sizes, the batches assignment to units, the sequencing of batches in each unit for each stage, and the initial and final times of the batches processed in each unit in order to minimize the campaign cycle time. With the aim of reducing the combinatorial complexity associated to the scheduling decisions, additional constraints are considered in order to eliminate equivalent symmetric solutions. Then, the scheduling approach through MPCs considering sequence-dependent changeover times for multistage batch plants with nonidentical parallel units is efficiently solved.

2 Problem description

The problem addressed in this article deals with a multiproduct batch plant where J denotes the set of processing stages that compose the plant and K the set of all units in the plant. K_j represents the set of nonidentical parallel batch units that operate out-of-phase in stage j, so $K = K_1 \cup K_2 \cup \ldots \cup K_{|J|}$.
A set I of products must be manufactured in the plant following the same sequence of stages. The total amount required of each product in the campaign, Q_i ($i \in$ I), which allows maintaining adequate stocks levels taking into account the estimated demands, is a model parameter. Q_i can be fulfilled with one or more batches, therefore an index b is introduced to denote the bth batch required to meet production of the corresponding product.
In each stage, there are not restrictions about parallel unit sizes and, therefore, different unit sizes are admitted. Then, V_k is used to denote the size of unit k. The processing time of each batch of product i in unit k, t_{ik}, and the size factor SF_{ij} that denotes the required capacity of units in stage j to produce one mass unit of final product i, are problem data.
Considering the demand of product i, the non-identical parallel unit sizes for each stage, the equipment utilization minimum rate for product i at each unit, denoted by α_{ik}, and the size factors of product i in each stage, the minimum and maximum numbers of batches required to fulfill the demand of product i can be calculated in order to ensure solution optimality. Thus, the minimum and maximum numbers of batches of product i for the campaign are calculated, respectively, as following:

$$NBC_i^{LOW} = \left\lceil \frac{Q_i}{B_i^{max}} \right\rceil \quad \text{and} \quad NBC_i^{UP} = \left\lceil \frac{Q_i}{B_i^{min}} \right\rceil \qquad \forall i$$

where $B_i^{max} = \min_{j \in J} \left\{ \max_{k \in K_j} \left\{ \frac{V_k}{SF_{ij}} \right\} \right\}$ and $B_i^{min} = \max_{j \in J} \left\{ \min_{k \in K_j} \left\{ \alpha_{ik} \frac{V_k}{SF_{ij}} \right\} \right\}$ are the maximum and minimum feasible batch sizes for product i. The upper bound for the number of batches of each product i in the campaign allows to propose a set of generic batches associated to that product, IB_i, where $|IB_i| = NBC_i^{UP}$.
Intermediate storage tanks are not allowed. Therefore, taking into account the configuration of the plant, there is no batch splitting or mixing, i.e. each batch is treated as a discrete entity throughout the whole process. It is assumed that a batch cannot wait in a unit after finishing its processing. Therefore, the ZW transfer policy between stages is adopted, i.e., after being processed in stage j, a batch b is immediately transferred to the next stage $j+1$. Besides, batch transfer times between units are assumed very small compared to process operation times and, consequently, they are included in the processing times.
Sequence-dependent changeover times, $c_{ii'k}$, are considered between consecutive batches processed in the same unit k, even of the same product. This transition time corresponds to the preparation or cleaning of the equipment to perform the following batch processing. It is necessary for various reasons: ensure products quality, maintain the equipment, safety reasons, etc.
For scheduling decisions, an asynchronous slot-based continuous-time representation has been used. The slots correspond to time intervals of variable length where batches will be assigned. In each slot l of a specific unit k at most one batch b of product i can be processed and, if no product is assigned to slot l, its length will be zero. The number of slots that must be postulated for unit k of stage j, denoted by L_{kj}, can be approximated considering the estimation on the maximum number of batches of each product at the campaign. Then, the number of slots postulated for all units of each stage is the same and it is given by:

$$L = \sum_{i \in I} NBC_i^{UP} \qquad \forall k, j$$

Although this value is an overestimation, a major approximation cannot be proposed taking into account that the parallel units are different and, on the other hand, the number and sizes of batches to be scheduled are optimization variables, unlike the most of scheduling approaches presented in the literature where they are considered as parameters. However, the lower bound on the number of batches of product i at the

campaign, NBC_i^{LOW}, strongly reduces the number of possible combinations and consequently improves the computational performance of the model.
As previously stated, the problem consists of solving simultaneously two decision levels often addressed sequentially. Through a holistic approach, the selection and sizing of batches of each product, the assignment of batches to units in each stage, the production sequence of assigned batches in each unit and initial and final processing times for batches that compose the campaign in each processing unit are jointly determined.

3 Mathematical formulation

3.1 Batches selection and sizing constraints
The number of batches of product *i* that must be manufactured in the campaign is a model variable. Then, a binary variable z_{ib} is introduced, which takes value 1 if batch *b* of product *i* is selected to satisfy the demand requirements of that product and 0 otherwise.
Let B_{ib} be the size of batch *b* of product *i* and Q_i the demand of product *i* that must be fulfilled, then:

$$Q_i = \sum_{b \in IB_i} B_{ib} \qquad \forall i \in I \tag{1}$$

Taking into account that the size of unit *k* denoted by V_k and the size factor SF_{ij} are model parameters, if batch *b* of product *i* is processed in unit *k* of stage *j* the following inequalities limit the size B_{ib} of batch *b* between the minimum and maximum processing capacities of unit *k*:

$$\alpha_{ik} \frac{V_k}{SF_{ij}} \le B_{ib} \le \frac{V_k}{SF_{ij}} \qquad \forall i \in I, b \in IB_i, k \in \{\text{units of stage } j \text{ used to process batch } b\} \tag{2}$$

where α_{ik} is the minimum filled rate required to process product *i* in unit *k*. Due to the units selected to process the batches of each product are optimization variables and their sizes are different, Eq. (2) must be expressed through a variable that indicates this selection, as it will see later.
Besides, without loss the generality and in order to reduce the number of alternative solutions, the selection of batches of a same product as well as the assigned sizes to them are made in ascending and descending numerical order, respectively, that is:

$$z_{ib+1} \le z_{ib} \qquad \forall i \in I, b \in IB_i, b+1 \in IB_i \tag{3}$$

$$B_{ib+1} \le B_{ib} \qquad \forall i \in I, b \in IB_i, b+1 \in IB_i \tag{4}$$

3.2 Assignment and Sequencing constraints
Selected batches must be assigned, in each stage, to specific slots in the units. Then, the binary variable Y_{bkl} is introduced, which takes value 1 if batch *b* is assigned to slot *l* in unit *k* and 0 otherwise. Although this variable is enough for formulating the scheduling problem, the binary variable X_{kl}, which specifies the slots set utilized in unit *k* for processing batches, will be also used in order to reduce the search space and, therefore, to improve the computational performance.
Logical relations can be defined among binary variables z_{ib}, X_{kl} and Y_{bkl}. In fact, if slot *l* of unit *k* is not utilized, then none of the proposed batches is processed in it. Moreover, if slot *l* of unit *k* is utilized, then only one of the proposed batches is processed in it. Then, the following constraint is imposed:

$$\sum_{i \in I} \sum_{b \in IB_i} Y_{bkl} = X_{kl} \qquad \forall j \in J, k \in K_j, 1 \le l \le L \tag{5}$$

On the other hand, if batch *b* of product *i* is selected (i.e. $z_{ib} = 1$), then this batch is processed, in each stage *j*, in only one slot of some of the available units at the stage. This condition is guaranteed by:

$$\sum_{k \in K_j} \sum_{1 \le l \le L} Y_{bkl} = z_{ib} \qquad \forall\, j \in J, i \in I, b \in IB_i \tag{6}$$

Without loss of generality and in order to reduce the search space, it is assumed that slots of each unit are consecutively used in ascending numerical order. Hence, the slots of zero length take place at the end of each unit. Eq. (7) establishes that for each unit k, slot $l+1$ is only used if slot l has been already allocated:

$$X_{kl} \ge X_{kl+1}, \qquad \forall\, j \in J, k \in K_j,\ 1 \le l \le L \tag{7}$$

Finally, variable Y_{bkl} allow correctly expressing the inequalities posed in (2) as:

$$\alpha_{ik} \frac{V_k}{SF_{ij}} Y_{bkl} \le B_{ib} \qquad \forall i \in I, b \in I, 1 \le l \le L \tag{8}$$

$$B_{ib} \le \frac{V_k}{SF_{ij}} + M_1 (1 - \sum_{1 \le l \le L} Y_{bkl}) \qquad \forall i \in I, b \in IB_i \tag{9}$$

where scalar M_1 is a sufficiently large number.

3.3. Timing constraints

Nonnegative continuous variables, TI_{kl} and TF_{kl}, are used to represent the initial and final processing times, respectively, of the proposed slots in each unit k. When slot l is not the last slot used in unit k of stage j for processing one batch, that is, if $Y_{b'kl+1}$ take value 1 for some b', final processing time TF_{kl} of slot l in unit k is constrained by:

$$TF_{kl} = TI_{kl} + \sum_{i \in I} \sum_{i' \in I} \sum_{b \in IB_i} \sum_{\substack{b' \in IB_{i'} \\ b \ne b'}} (t_{ik} + c_{ii'k}) Y_{bkl} Y_{b'kl+1} \qquad \forall\, j \in J, k \in K_j, 1 \le l < L \tag{10}$$

A nonnegative variable $YY_{blb'l'k}$ is defined to eliminate the bilinear products, which takes value 1 if $Y_{bkl} = 1$ and $Y_{b'kl+1} = 1$, and 0 otherwise, so (10) is represent using Big-M expressions:

$$TF_{kl} - TI_{kl} - \sum_{i \in I} \sum_{i' \in I} \sum_{b \in IB_i} \sum_{\substack{b' \in IB_{i'} \\ b \ne b'}} (t_{ik} + c_{ii'k}) YY_{blb'l+1k} \ge M_2 (X_{kl+1} - 1) \quad \forall\, j \in J, k \in K_j, 1 \le l < L \tag{11a}$$

$$-TF_{kl} + TI_{kl} + \sum_{i \in I} \sum_{i' \in I} \sum_{b \in IB_i} \sum_{\substack{b' \in IB_{i'} \\ b \ne b'}} (t_{ik} + c_{ii'k}) YY_{blb'l+1k} \ge M_2 (X_{kl+1} - 1) \quad \forall\, j \in J, k \in K_j, 1 \le l < L \tag{11b}$$

On the other hand, when the sequence of slots used in unit k is 1, 2, ... l, i.e. slot l is the last slot used at unit k of stage j to process some batch, taking into account that the campaign can be cyclical repeated over a time horizon, the final processing time TF_{kl} is calculated considering the changeover time required for processing the batch assigned to slot 1 in unit k of stage j. Constraints analogous to (11a) and (11b) are posed for this case.

Constraints to avoid the overlapping between the processing times of different slots in a unit as well as to match the initial times of empty slots with the final time of the previous slot are added to the formulation. In order to assure ZW transfer policy, constraints of Big-M type are included, depending if slot l is or is not the last slot used at unit k for processing one batch. Due to space reasons, this set of constraints is not provided in this manuscript, but interested readers can request it to authors.

Finally, taking into account that slots of each unit are used in ascending numerical order, the expression for the cycle time of the campaign, CT, is given by:

$$CT \ge TF_{kL} - TI_{k1}, \qquad \forall j \in J, k \in K_j \tag{12}$$

3.4. Objective function

The problem goal is to minimize the cycle time of the production campaign that fulfills the demands requirements, subject to previous constraints.

4 Example

The considered batch plant consists of three stages with nonidentical parallel units with known sizes that operate out-of-phase, as is illustrated in Figure 1. Available units at each stage are denoted by the sets: $K_1 = \{1\}$, $K_2 = \{2, 3\}$, and $K_3 = \{4, 5\}$, respectively. Products A, B, and C have to be processed through all stages before being converted into final products. The required amounts in the campaign are $Q_A = 10500$, $Q_B = 6000$ and $Q_C = 9500$. Data on processing times and size factors of each product are shown in Table 1, while the sequence-dependent changeover times are given in Table 2.
Considering the non-identical parallel unit sizes at each stage, the size factors for each product in each stage and assuming that the equipment utilization minimum rate is 0.50 for all products and equipment items, the minimum feasible batch sizes for products A, B and C are:

$$B_A^{min} = 0.5\,max\{5714\,\text{kg},\ 5000\,\text{kg}, 5000\,\text{kg}\} = 2857\,\text{kg},$$

$$B_B^{min} = 0.5\,max\{6666\,\text{kg},\ 4285\,\text{kg}, 5555\,\text{kg}\} = 3333\,\text{kg},$$

$$B_C^{min} = 0.5\,max\{5714\,\text{kg},\ 4615\,\text{kg},\ 4545\,\text{kg}\} = 2857\,\text{kg}.$$

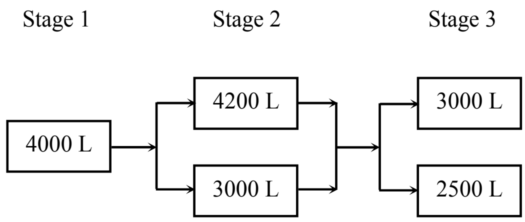

Figure 1. Plant structure

Table 1. Processing times and size factors of products

Product	Processing time: t_{ik} (h)					Size factor: SF_{ij} (L/kg)		
	Stage 1	Stage 2		Stage 3		Stage 1	Stage 2	Stage 3
i	1	2	3	4	5	$k = 1$	$k = 2, 3$	$k = 4, 5$
A	14	25	20	7	6	0.70	0.60	0.50
B	16	18	18	5	3	0.60	0.70	0.45
C	12	15	12	4	3	0.70	0.65	0.55

Table 2. Sequence-dependent changeover times

Product	Sequence-dependent changeover time: $c_{ii'k}$ (h)								
	Stage 1			Stage 2			Stage 3		
	$k = 1$			$k = 2, 3$			$k = 4, 5$		
i	A	B	C	A	B	C	A	B	C
A	0	0.5	0.3	0.25	0.3	0.4	0	0.6	0.6
B	0.8	0	0.6	2.2	0.25	0.8	0.8	0	0.8
C	1	0.5	0	3	1.5	0.25	2	1.5	0

Then, considering the campaign demands for all products, the maximum number of batches of each product at the campaign is four, two and four for products A, B and C, respectively.
Thus, the sets of proposed batches are {b1, b2, b3, b4}, {b5, b6}, and {b7, b8, b9, b10} for products A, B and C, respectively, and consequently a total of ten batches must be postulated to guarantee the global optimality of the solution. Also, the maximum feasible batch sizes for all products allow determining the minimum number of batches of every product at the campaign. In this case, the maximum feasible batch sizes for all products are:

$$B_A^{max} = min\{5714\,\text{kg},\ 7000\,\text{kg},\ 6000\,\text{kg}\} = 5714\,\text{kg}$$

$$B_B^{max} = min\{6666\,\text{kg},\ 6000\,\text{kg},\ 6666\,\text{kg}\} = 6000\,\text{kg}$$

$$B_C^{max} = min\{5714\,\text{kg},\ 6461\,\text{kg},\ 5454\,\text{kg}\} = 5454\,\text{kg}$$

then, the required minimum number of batches for products A and C is two, while for product B is one. The model under these assumptions comprises 52052 constraints, 9167 continuous variables and 555 binary variables. It was implemented and solved using GAMS, via CPLEX 12.5 solver, in 42.77 CPU seconds with a 0% of optimality gap. The optimal campaign cycle time is equal to 70.4 hours and it involves two batches of product A (b_1, b_2), one of B (b_5), and two of C (b_7, b8), i.e. the demands of all products are fulfill with the minimum number of batches. The optimal production sequence obtained in each batch unit for the different stages, considering sequence-dependent changeover times, is illustrated in the Gantt chart of Figure 2. Taking into account that the optimal campaign is cyclically repeated over a time horizon, the changeover times between products processed in the last and first slot of each unit must be included in the optimization in order to achieve the accurate overlap of successive campaigns. For this example, as it can be seen from Figure 2, changeover times between pairs of campaigns are: c_{AC1} = 0.3 h for unit of stage 1; $c_{AC2} = c_{AC3}$ = 0.4 h for units of stage 2; and $c_{AC4} = c_{AC5}$ = 0.6 h for units of stage 3. Batches b_1 and b_2 satisfy the total required demand of product A with sizes of 5500 kg and 5000 kg, respectively; batch b_5 with size equal to 6000 kg is only selected to accomplish the campaign demand of product B; while batches b_7 and b_8 are required to achieve the production of C with sizes equal to 4955 kg and 4545 kg, respectively. The capacities used in each unit of the different stages for processing the selected batches are resumed in Table 3. The batches that reach the maximum capacities are highlighted in boxes shaded in gray. Batch b_2 of product A is processed in units 1, 3 and 5 and its size is the maximum possible to be processed in units 3 and 5 of stages 2 and 3, respectively. Then, batch b_1 fulfills the required amount of that product occupying approximately 96%, 79% and 92% of the capacity of units 1, 2 and 4, respectively. Batch b_5 of product B is processed in units 1, 2 and 4 and its size is the maximum possible to be processed in unit 2 of stage 2. On the other hand, two batches of product C are processed for meeting its demand. Batch b_8 is processed in units 1, 3 and 5 using 80%, 98.5% and 100% of their capacities, respectively; while batch b_7 fulfills the required amount of that product in the campaign.

5 Conclusions

In this work, the optimal production planning of multistage batch plants with nonidentical parallel units that operate in campaign-mode is faced. Scheduling is modeled according to campaign-based operation mode in such way that the campaign cycle time minimization is an appropriate optimization criterion. Sequence-dependent changeover times are considered for each ordered pair of products in each unit of the different stages.
Taking into account the complexity of the simultaneous involved decisions, some additional constraints that eliminate equivalent symmetric solutions maintaining the model generality are considered, in order to reduce the search space and therefore improve the computational performance. Also, various equations are reformulated in order to keep the problem linear and assure the global optimality of the solution. Through the example the capabilities of the proposed formulation are shown. With the proposed

formulation, an interesting problem has been solved. Many times, in made-to-stock contexts, the campaign-based operation mode is an appropriate alternative that allows taking advantage of the available resources with an ordered production management. The proposed model simultaneously solves lot-sizing and scheduling problems in reasonable computing time. Thus, this approach can be applied in real production systems that operate in campaign-mode taking into account the assumed suppositions as far as different unit sizes, changeovers, etc.

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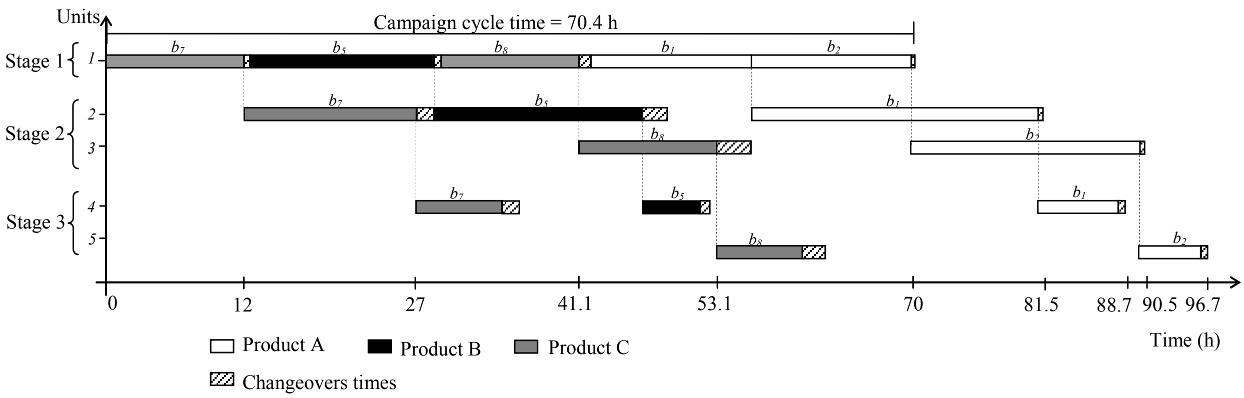

Figure 2. Gantt chart of the production campaign

Table 3. Capacities used in each unit of each stage

		Stage 1	Stage 2		Stage 3	
Product	Batch	*k* = 1	*k* = 2	*k* = 3	*k* = 4	*k* = 5
A	b_1	3850	3300		2750	
	b_2	3500		3000		2500
B	b_5	3600	4200		2700	
C	b_7	3468.2	3220.4		2725	
	b_8	3181.8		2954.5		2500