Simultaneous planning and scheduling of multistage multiproduct batch plants with non-identical parallel units and sequence-dependent changeovers

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***Abstract*—A mixed integer linear programming model for the simultaneous planning and scheduling of multistage multiproduct batch plants in a multiperiod context is presented in this work. The batch plant is composed by a set of stages with parallel units of different sizes. The scheduling decisions are modeled considering mixed product campaign-based operation mode and sequence-dependent changeover times. Thus, for each time period, the number of batches of each product in the campaign, the batch assignment to units in each stage and its sequencing, and the number of repetitions of the campaign are jointly determined. Also, decisions about production and storage of different products, and raw material consumption and storage are taken into account. The approach capabilities are highlighted through an example.**

Keywords—mixed product campaign; non-identical parallel units; changeovers; multiperiod planning; mixed integer linear programming.

I. Introduction

The increasing product customization and diversification in the chemical industry have led to the installation (or retrofit) of facilities where different products have to share limited resources (equipment units and utilities) and which can be operated in multiple modes. The flexibility of this type of plants, called multiproduct facilities, improves the resource utilization and allows lower inventory costs and better reaction to demand fluctuations. However, these advantages can only be materialized if the production is planned well, a task which is hard mainly due to the increased flexibility, and, thus, multiplicity of solutions [1].

A number of researchers have proposed scheduling as well as integrated production planning and scheduling approaches in the area of process system engineering (PSE). The two problems are interdependent since the solution of production planning (production targets) is input to scheduling, and the production capacity constraints in production planning depend on the scheduling solution [2]. Reference [3] presents an excellent review of methodologies and solution strategies of scheduling problem, while [4], [5] and [6] provide overviews of production planning and scheduling approaches.

In this work, a mixed integer linear programming (MILP) model for the simultaneous planning and scheduling of a batch plant over different time periods is proposed. Deterministic variations in prices, product demands limits, costs, and raw materials availability due to seasonal or market fluctuations are included in this approach. The operation policy based on mixed product campaigns (MPCs) is adopted in order to reduce idle times, increase unit utilization, and avoid inventory buildups of materials. The objective is to determine the detailed production planning scheme, inventory levels, and raw material consumption that allow fulfilling customer demand limits at the maximum benefits.

The proposed formulation represents a suitable tool for decision making in supporting the plant management.

II. Problem definition

The problem addressed deals with a multiproduct batch plant that processes I products during T time periods of duration H_t. All products follow the same production sequence throughout J batch processing stages and they are produced using C raw materials. K_j represents the set of non-identical parallel batch units that operate out-of-phase in stage j. Since different unit sizes are admitted in each stage, V_{kj} is used to denote the size of unit k of stage j. The processing time of each product i in unit k, t_{ik}, and the size factor SF_{ij} that denotes the required capacity of units in stage j to produce one mass unit of final product i, are problem data. Sequence-dependent changeover times, $c_{ii'k}$, are considered between consecutive batches processed in the same unit k, even of the same product. This transition time corresponds to the preparation or cleaning of the equipment to perform the following batch processing.

Intermediate storage tanks are not allowed. Besides, it is assumed that a batch cannot wait in a unit after finishing its processing. Therefore, the Zero Wait (ZW) transfer policy between stages is adopted. This policy assumes that a batch, after finishing its processing at a stage, must be transferred immediately to the next stage.

For each product i, lower and upper bounds on its demands in every period t, DE_{it}^{L} and DE_{it}^{U}, are known. The amounts of raw materials consumed are determined by mass balances with a given parameter F_{cit} that accounts for the process conversion of raw material c to produce product i during period t. Costs and availability of raw materials as well as prices of final products vary from period to period and they are assumed to be known. Maximum available storage capacities are problem data and at the beginning of the global time horizon, the initial inventories of both raw material and product, IM_{i0} and IP_{i0}, are assumed to be given.

During each time period, the plant operates in MPC mode, i.e. the production campaign, composed by a set of batches of the different products manufactured in this period, is cyclically repeated over H_t. For product i, a set of generic batches associated to that product, IB_i, is proposed and the size of this set is the maximum number of batches in the campaign. An asynchronous slot-based continuous-time representation for modeling the scheduling decisions is employed [7]. The slots correspond to time intervals of variable length where batches will be assigned. In each slot l of a specific unit k at most one batch b of product i can be processed and, if no product is assigned to slot l, its length will be zero.

Then, the problem consists of determining for each time period t: (1) the amounts of product i to be produced Q_{it}, raw material c to be used for production, RM_{ct}, purchased raw material c during the period, C_{ct}, and wasted raw material, RW_{ct}; (2) the MPC composition, i.e. the number of batches of each product in the campaign, and the size of each batch of product i elaborated in the campaign of period t; (3) the assignment of batches to units in each stage, the production sequence on each unit, initial and final processing times for the batches in each unit and the campaign cycle time, CTC_t; (4) the number of times that the campaign is cyclically repeated over the time horizon H_t, denoted by NN_t; (5) the levels of both final product, IP_{it}, and raw material inventories, IM_{it}; (6) the wastes due to the expired product shelf life, PW_{it}; and (7) the total sales of each product i elaborated, QS_{it}; in order to maximize the net benefit.

III. MODEL FORMULATION

The model basically considers two sets of constraints, which are summarized below:

A. *Production planning constraints*

These constraints allow determining, at each time period, the amount of raw materials purchased and used for producing each product, the total production, the levels of raw material and final product inventories, and the amount of sold products. The equations description is omitted due to space reasons.

B. *Scheduling constraints*

1) Batching constraints: The number of batches of product i that must be manufactured in the campaign of period t is a model variable. Then, a binary variable z_{ibt} is introduced, which takes value 1 if batch b of product i is selected to satisfy the production level of that product at the period t and 0 otherwise.

Let B_{ibt} be the size of batch b of product i elaborated in period t and Q_{it} the total amount of product i produced in that time period. Then, taking into account that the MPC will be cyclically repeated NN_t times over the time period t:

$$Q_{it} = \sum_{b \in IB_i} B_{ibt} NN_t \qquad \forall i,t \qquad (1)$$

Due to B_{ibt} and NN_t are optimization variables, (1) is reformulated to avoid non linearities. Discrete variable NN_t can be expressed using a 2-based representation as:

$$NN_t = \sum_{m=0}^{M_t} c_{mt} 2^m, \qquad \forall t \qquad (2)$$

Parameter M_t = ceil$\left(log_2 (NN_t^{UP} + 1) - 1\right)$, where ceil is a function that rounds the argument to the next integer, NN_t^{UP} is the maximum number of times that the campaign can be cyclically repeated over the time period t and c_{mt} are binary variables.

In particular, if production at the period t is null, all binary variables c_{mt} take value zero:

$$\sum_i \sum_b z_{ibt} \geq c_{mt}, \qquad \forall m,t \qquad (3)$$

Then, replacing (2) into (1), the following constraint is hold:

$$Q_{it} = \sum_{b \in IB_i} \sum_{m=0}^{M_t} c_{mt} B_{ibt} 2^m \qquad \forall i,t \qquad (4)$$

Bilinear terms in (4) are eliminated defining a non negative continuous variable, w_{ibmt}, which is equal to B_{ibt} if c_{mt} take value 1, and 0 otherwise. So (4) is represented by:

$$Q_{it} = \sum_{b \in IB_i} \sum_{m=0}^{M_t} 2^m w_{ibmt} \qquad \forall i,t \qquad (5)$$

Besides, the following constraints are imposed, where M_1 is a sufficiently large number that makes the constraint redundant when c_{mh} takes value 0:

$$w_{ibmt} - B_{ibt} \geq M_1 (c_{mt} - 1) \qquad \forall i, b \in IB_i, m, t \qquad (6)$$

$$w_{ibmt} \leq B_{ibt} \qquad \forall i, b \in IB_i, m, t \qquad (7)$$

$$w_{ibmt} \leq B_i^{UP} c_{mt} \qquad \forall i, b \in IB_i, m, t \qquad (8)$$

Taking into account that the size of unit k of stage j denoted by V_{kj} and the size factor SF_{ij} are model parameters, if batch b of product i is processed in unit k of stage j during period t, the following inequalities limit the size B_{ibt} of batch b between the minimum and maximum processing capacities of unit k:

$$\alpha_{ik} \frac{V_{kj}}{SF_{ij}} \leq B_{ibt} \leq \frac{V_{kj}}{SF_{ij}} \qquad \forall i \in I, b \in IB_i, t \qquad (9)$$
$$k \in \{\text{units of stage } j \text{ used to process batch } b\}$$

where α_{ik} is the minimum filled rate required to process product i in unit k. Due to the units selected to process the batches of each product are optimization variables and their sizes are different, (9) must be expressed through a variable that indicates this selection, as it will see later.

Besides, without loss the generality and in order to reduce the number of alternative solutions, the selection of batches of a same product as well as the assigned sizes to them are made in ascending and descending numerical order, respectively, that is:

$$z_{ib+1t} \le z_{ibt} \qquad \forall i \in I, b \in IB_i, b+1 \in IB_i, t \qquad (10)$$

$$B_{ib+1t} \le B_{ibt} \qquad \forall i \in I, b \in IB_i, b+1 \in IB_i, t \qquad (11)$$

2) Assignment and sequencing constraints: For each period t, selected batches must be assigned, in each stage, to specific slots in the units. Then, the binary variable Y_{bklt} is introduced, which takes value 1 if batch b is assigned to slot l in unit k in period t and 0 otherwise. Although this variable is enough for formulating the scheduling problem, the binary variable X_{kl}, which specifies the slots set utilized in unit k for processing batches, will be also used in order to reduce the search space and, therefore, to improve the computational performance.

Logical relations can be defined among binary variables z_{ibt}, X_{klt} and Y_{bklt}. In fact, if slot l of unit k is not utilized in period t, then none of the proposed batches is processed in it. Moreover, if slot l of unit k is utilized, then only one of the proposed batches is processed in it. Then, the following constraint is imposed:

$$\sum_{i \in I} \sum_{b \in IB_i} Y_{bklt} = X_{klt} \qquad \forall\ j, k \in K_j, l, t \qquad (12)$$

On the other hand, if batch b of product i is selected (i.e. z_{ibt} = 1), then this batch is processed in only one slot of some of the available units at each stage j. This condition is guaranteed by:

$$\sum_{k \in K_j} \sum_{1 \le l \le L} Y_{bklt} = z_{ibt} \qquad \forall\ j, i, b \in IB_i, t \qquad (13)$$

Without loss of generality and in order to reduce the search space, it is assumed that slots of each unit are consecutively used in ascending numerical order. Hence, the slots of zero length take place at the end of each unit. Eq. (14) establishes that for each unit k, slot $l+1$ is only used if slot l has been already allocated:

$$X_{klt} \ge X_{kl+1t}, \qquad \forall\ j, k \in K_j, l, t \qquad (14)$$

Finally, variable Y_{bklt} allows correctly expressing the inequalities posed in (9) as:

$$\alpha_{ik} \frac{V_{kj}}{SF_{ij}} Y_{bklt} \le B_{ibt} \quad \forall i, b \in IB_i, j,\ k \in K_j, l, t \qquad (15)$$

$$B_{ibt} \le \frac{V_{kj}}{SF_{ij}} + M_3 (1 - \sum_{1 \le l \le L} Y_{bklt}) \qquad \forall i, b \in IB_i, j, k \in K_j, t \qquad (16)$$

where scalar M_3 is a sufficiently large number.

3) Timing constraints: Nonnegative continuous variables, TI_{klt} and TF_{klt}, are used to represent the initial and final processing times, respectively, of the proposed slots in each unit k during period t. When slot l is not the last slot used in unit k of stage j for processing one batch, that is, if $Y_{b'kl+1t}$ take value 1 for some b', final processing time TF_{klt} of slot l in unit k at period t is constrained by:

$$TF_{klt} = TI_{klt} + \sum_{i \in I} \sum_{i' \in I} \sum_{b \in IB_i} \sum_{\substack{b' \in IB_{i'} \\ b \ne b'}} (t_{ik} + c_{ii'k}) Y_{bklt} Y_{b'kl+1t} \qquad \forall\ j, k \in K_j, l, t \qquad (17)$$

A nonnegative variable $YY_{blb'l'kt}$ is defined to eliminate the bilinear products, which takes value 1 if Y_{bklt} =1 and $Y_{b'kl+1t}$ =1 and 0 otherwise, so (17) is represent using Big-M expressions.

On the other hand, when the sequence of slots used in unit k is 1, 2, ... l, i.e. slot l is the last slot used at unit k of stage j to process some batch, taking into account that the campaign can be cyclical repeated over time period t, the final processing time TF_{klt} is calculated considering the changeover time required for processing the batch assigned to slot 1 in unit k of stage j. Constraints of type Big-M analogous to those required to represent (17) are posed for this case.

Constraints to avoid the overlapping between the processing times of different slots in a unit as well as to match the initial times of empty slots with the final time of the previous slot are considered in the formulation. In order to assure ZW transfer policy, constraints of Big-M type are included, depending if slot l is or is not the last slot used at unit k for processing one batch. Due to space reasons, this set of constraints is not provided in this manuscript, but interested readers can request it to authors.

Finally, taking into account that, for all periods, slots of each unit are used in ascending numerical order, the expression for the cycle time of the campaign, CTC_t, is given by:

$$CTC_t \ge TF_{kLt} - TI_{k1t}, \qquad \forall j, k \in K_j, t \qquad (18)$$

Then, as the campaign is cyclically repeated NN_t times along the time period H_t, the following inequality is imposed:

$$CTC_t NN_t \le H_t, \qquad \forall t \qquad (19)$$

Equation (2) allows transforming (19) in the nonlinear inequality:

$$\sum_{m=0}^{M_t} 2^m c_{mt} CTC_t \le H_t, \qquad \forall t \qquad (20)$$

Then, a nonnegative variable ww_{mt} is defined to eliminate the bilinear products, which is equal to CTC_t if c_{mt} =1 and 0 otherwise.

C. Objective function

The problem goal is to maximize the net benefit, which is calculated by the difference between the revenue due to product sales and the overall costs, with the latter consists of the expenses due to raw materials, inventories, operation, wastes due to the expired raw material and products shelf life.

IV. Example

The considered batch plant consists of three stages with two non-identical parallel units operating out-of-phase on stage 2. The units at each stage are denoted by the sets: K_1 = {1}, K_2 = {2, 3}, and K_3 = {4}, respectively. Units sizes are 4000 L in stage 1, 4200 L and 3000 L in stage 2, and 3000 L in stage 3. The plant produces 3 products (A, B and C) with 2 different raw materials (C1 and C2). A global time horizon of 1 month (576 h) with 4 equal time periods of 1 week each (H_t = 144 h) is considered.

Data on processing times, size factors and sequence-dependent changeover times are shown in Table I. Prices of raw materials and final products, and maximum bounds on demand forecasts over each period, are given in Table II. Minimum product demands in each period are assumed as 50% of maximum product demands. Due to space reasons, data on conversion factors, products and raw materials lifetimes in time periods, and inventory costs of both final products and raw materials were not reported. However, they are available for everyone who requests them.

Considering the unit sizes at each stage, the size factors for each product in each stage, and assuming that the equipment utilization minimum rate is 0.50 for all products and equipment items, the minimum feasible batch sizes for products A, B and C are:

$$B_A^{min} = 0.5\,max\{5714\,\text{kg},\ 5000\,\text{kg}, 6000\,\text{kg}\} = 3000\,\text{kg}$$

$$B_B^{min} = 0.5\,max\{6666\,\text{kg},\ 4285\,\text{kg}, 6666\,\text{kg}\} = 3333\,\text{kg}$$

$$B_C^{min} = 0.5\,max\{5714\,\text{kg},\ 4615\,\text{kg},\ 5454\,\text{kg}\} = 2857\,\text{kg}\ .$$

For each period, the number of batches of product i in the composition of the campaign is upper bounded by 3. Then, the sets of batches proposed for each product are: IB_A={b_1, b_2, b_3}, IB_B={b_4, b_5, b_6} and IB_C={b_7, b_8, b_9}. The upper bound for the variable representing the number of repetitions of the campaign over H_t, NN_t, is proposed considering one extreme type of campaign that can be arisen in period, namely, that with minimum cycle time. For this example, the upper bound for variable NN_t is fixed to 12, i.e. three binary variables have been defined for the 2-based representation of that variable. In order to avoid undesirable combinations for the value of NN_t, the following constraints are added to the formulation: $c_{3t} + c_{2t} + c_{1t} \leq 2$; $c_{3t} + c_{2t} + c_{0t} \leq 2,\ \forall\ t$.

The model under these assumptions comprises 70296 linear constraints, 16396 continuous variables, and 1092 binary variables. It was implemented and solved using GAMS, via CPLEX 12.1 solver, in 1853.14 CPU seconds with a 0% of optimality gap.

For each period, the amounts of final products produced and sold, amounts of raw materials purchased for producing all products, and the inventories levels of both raw materials and products, are summarized in Table III.

Both raw materials are purchased in periods where costs are the lowest ones, except C2 due to raw material and final product inventories levels are null at the beginning of the planning horizon and both raw material are required to satisfy the production of period 1. For raw material C1, the extra material purchased in period 1 is kept as inventory for fulfilling production in the two next periods. Analogously, for raw material C2, the extra material purchased in periods 1 and 3 are kept as inventory for production in subsequent periods. For each product, extra amounts are produced in some periods, which are kept as inventory to satisfy maximum demands in subsequent periods.

For each period t, the batches of each product involved in the optimal production campaign, batch sizes, the campaign cycle time and the number of times that it is repeated over the time period are depicted in Table IV. Figs. 1 to 4 illustrate the production sequence in the different stages for all time periods. Since product C is not produced in period 4, the campaign only consists of one batch of each of the other products, which is sufficient to meet the production plan in this period. The economic results for this example are summarized in Table V.

V. Conclusions

In this work, the optimal production planning and scheduling of multistage batch plants with non-identical parallel units that operate in campaign-mode is addressed. Scheduling decisions are modeled according to this operation mode. So, in each period, the campaign cycle time must be calculated in order to achieve the appropriate overlapping of them. Sequence-dependent changeover times are considered for each ordered pair of products in each unit of the different stages.

Taking into account the complexity of the simultaneous involved decisions, some additional constraints that eliminate equivalent symmetric solutions maintaining the model generality are considered, in order to reduce the search space and therefore improve the computational performance. Also, various equations are reformulated in order to attain a MILP model and assure the global optimality of the solution. Through the example the capabilities of the proposed formulation are shown.

With the proposed model, the operation management and production planning, which are common activities at the plant floor, can be controlled. Moreover, it allows making different decisions, like the forecast of material requirement, the inventory management of raw materials and final products, the distribution policy determination, etc.

TABLE I. PROCESS DATA

Product i	Processing times: t_{ik} (h)				Size factors: SF_{ij} (L/kg)			Sequence-dependent changeover times: $c_{ii'k}$ (h)								
	Stage 1	*Stage 2*		*Stage 3*	*Stage 1*	*Stage 2*	*Stage 3*	*Stage 1*			*Stage 2*			*Stage 3*		
	1	***2***	***3***	4	***1***	***2, 3***	***4***	***A***	***B***	***C***	***A***	***B***	***C***	***A***	***B***	***C***
A	13	24	20	7	0.70	0.60	0.50	0.0	0.5	0.3	0.25	0.3	0.4	0.0	1.0	0.0
B	16	18	18	5	0.60	0.70	0.45	0.0	0.0	1.0	1.2	0.25	0.8	0.5	0.0	0.8
C	12	15	12	4	0.70	0.65	0.55	1.5	0.0	0.0	1.5	0.5	0.25	2.25	1.0	0.25

TABLE II. PRICES AND DEMAND BOUNDS

Period t	Raw material costs ($/kg)		Products prices ($/kg)			Maximum demands (× 10^3 kg)		
	C1	***C2***	***A***	***B***	***C***	***A***	***B***	***C***
1	0.5	1.4	2.05	2.6	2.0	15.0	16.5	9.0
2	1.0	0.6	2.25	2.6	2.20	16.5	25.5	15.0
3	1.0	0.6	2.25	2.4	2.20	13.15	15.0	7.5
4	0.5	1.8	2.05	2.4	2.0	15.0	20.0	16.2

TABLE III. OPTIMAL PRODUCTION PLAN FOR EACH PERIOD

Period t	Product A (× 10^3 kg)			Product B (× 10^3 kg)			Product B (× 10^3 kg)			Raw material C1 (× 10^3 kg)		Raw material C2 (× 10^3 kg)	
	Q_{it}	QS_{it}	IP_{it}	Q_{it}	QS_{it}	IP_{it}	Q_{it}	QS_{it}	IP_{it}	C_{ct}	IM_{ct}	C_{ct}	IM_{ct}
1	15.0	15.0	0.0	18.0	16.5	1.5	15.0	9.0	6.0	112.71	76.734	128.26	69.18
2	19.65	16.5	3.15	24.0	25.5	0.0	10.9	15.0	1.9	0.0	35.27	0.0	0.0
3	10.0	13.15	0.0	15.0	15.0	0.0	21.8	7.5	16.2	0.0	0.0	101.32	46.5
4	15.0	15.0	0.0	20.0	20.0	0.0	0.0	16.2	0.0	27.50	0.0	0.0	0.0

TABLE IV. OPTIMAL PRODUCTION CAMPAIGN FOR EACH TIME PERIOD

Period t	Batch sizes for product A (kg)			Batch sizes for product B (kg)			Batch sizes for product C (kg)			Campaign cycle time: CTC_t (h)	Number of repetitions: NN_t
	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9		
1	5000	0	0	6000	0	0	4990	0	0	41.3	3
2	5000	4825	0	6000	6000	0	5455	0	0	70.8	2
3	5000	0	0	4166	3334	0	5455	5455	0	70.3	2
4	3750	0	0	5000	0	0	0	0	0	29.5	4

TABLE V. ECONOMIC EVALUATION RESULTS

Description	Optimal value ($)
Sales income	419812.50
Raw material cost	255002.50
Raw material inventory cost	327.87
Product inventory cost	413.35
Operating cost	18435.00
Waste disposal cost	0.00
Total	145633.78

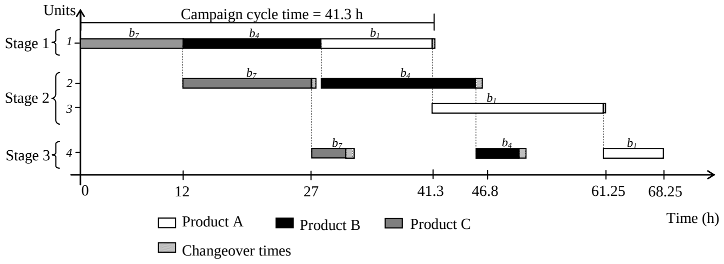

Fig. 1. Gantt chart of the optimal MPC for period 1.

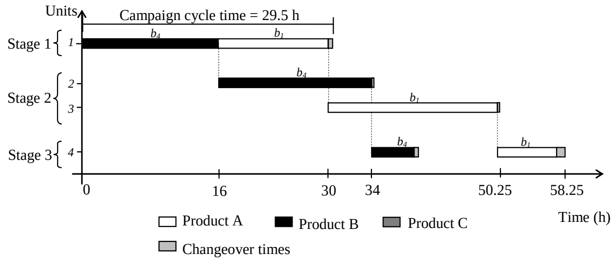

Fig. 2. Gantt chart of the optimal MPC for period 4.

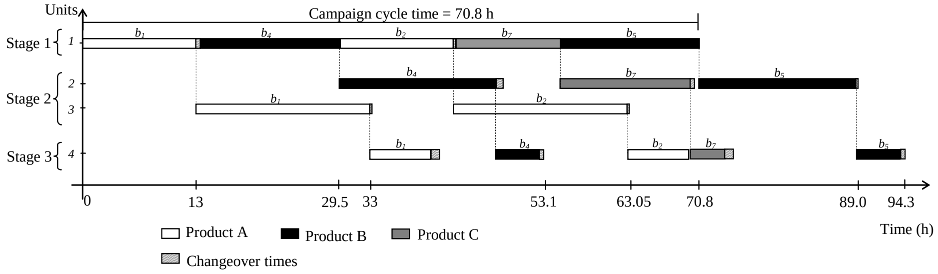

Fig. 3. Gantt chart of the optimal MPC for period 2.

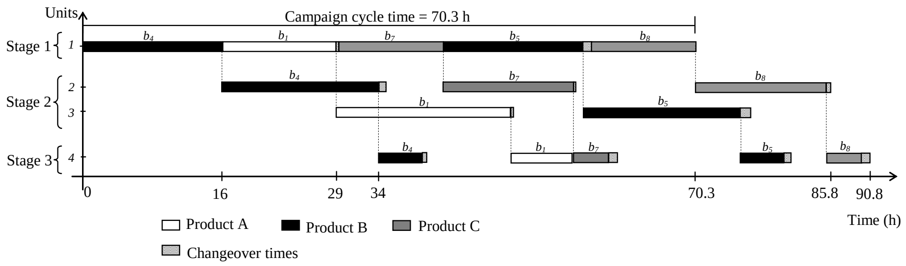

Fig. 4. Gantt chart of the optimal MPC for period 3.

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References

[1] A. Sundaramoorthy, C.T. Maravelias, "A general framework for process scheduling" AIChE J., vol. 57(3), pp. 695-710, 2011.

[2] G.M. Kopanos, L. Puigjaner, C.T. Maravelias, "Production planning and scheduling of parallel continuous processes with product families." Ind. Eng. Chem. Res., vol. 50(3), pp. 1369-1379, 2011.

[3] C.A. Méndez, J. Cerdá, I.E. Grossmann, I. Harjunkoski, and M. Fahl, "State-of-the-art of optimization methods for short-term scheduling of batch processes." Comput. Chem. Eng., vol. 30, pp. 913–946, 2006.

[4] N. Shah, "Single and multisite planning and scheduling: current status and future challenge." Proceedings of the Third International Conference of the Foundations of Computer-Aided Process Operations, Snowbird, Utah, pp 75-90, 1998.

[5] J. Kallrath, "Planning and scheduling in the process industry. " OR Spectrum, vol. 24, pp. 219–250, 2002.

[6] C.T. Maravelias, C. Sung, "Integration of production planning and scheduling: Overview, challenges and opportunities." Comput. Chem. Eng., vol. 33, pp. 1919–1930, 2009.

[7] Y. Fumero, G. Corsano, J. M. Montagna, "Scheduling of multistage multiproduct batch plants operating in a campaign-mode". Ind. Eng. Chem. Res., vol. 5, pp. 3988−4001, 2012.