Optimum Multiobjective Regulator with Variable Gain Matrix Applied to an Industrial Process

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***Abstract*—In this paper, it is presented the design and tuning of a robust multi-objective regulator, with variable gain matrix through Linear Matrix Inequalities (LMI). The tuning presented here guarantees for an uncertain model under polytopic representation to satisfy simultaneously multiple objectives such as, asymptotic stability, minimization of the $\mathcal{H}_2$-norm, robustness and restrictions imposed on the manipulated variable.**

Finally, as an application example, this tuning is applied to a Continuous Stirred Tank Reactor (CSTR) typical of the chemical industry.

***Index Terms*—LMI, Optimal Control Multiobjectives, LTV Systems, Variable Gain Matrix.**

I. Introduction

It is well known, in classical process control [1]–[3] among others, that the non-linear nature and the extremely slow dynamic responses of the industrial processes forces to tune controllers/regulators that guarantee a certain robustness to the control system, in order to overcome the uncertainties in the modeling. The situation is even more complicated when, in addition to the above, the restrictions of the process are added.

The response of traditional process control to this problem is to tune industrial controllers, using traditional techniques, resulting in extremely conservative dynamic responses.

Some authors such as Cao and Fang [4] propose a new approach by analyzing Lyapunov's function dependent on parameters related to uncetainty and saturation, to reduce conservatism in the stability analysis for polytopic systems subject to saturation input, via LMI. On the other hand, Henrion et al. [5] studied the output feedback robust stabilization of uncertain linear systems with saturating controls and, Henrion and Tarbouriech [6] studied the LMI relaxations to achieve the objectives mentioned before. It is important to remark that, the mentioned works implement a gain matrix by mean of offline calculus.

To achieve the desired objectives with less conservative results than those obtained by traditional process control techniques, it is proposed in this work the design of the multi-objectives optimal regulator by full-state feedback with variable gain matrix. Furthermore, the design specifications are written by means of Linear Matrix Inequalities (LMI), the uncertainties system are modeled through a polytopic model, and implementing a variable feedback gain vector which is calculated by time intervals.

In order to highlight the benefits of the proposed regulator, the temperature control of a Continuous Stirred Tank Reactor (CSTR) is taken as an example.

This work is organized as detailed below. In Section II, the basic concepts of the LQR regulator via LMI are presented. The analysis applies to a nominal LTI system as well as to uncertain systems with polytopic representation. In Section III, the design proposal of the nominal LQR controller for generalized systems with restrictions is introduced. At the end of this section, a discretization procedure to obtain a state feedback with variable gain matrix is presented. In Section IV, a comparison is presented through numerical simulations that show the benefits of the proposed regulator. Finally, in Section V, the conclusions of this work are presented.

II. Preliminary concepts

A. *Optimal Regulator. $\mathcal{H}_2$-Norm Minimization*

According to the optimal control theory, it is known that the control signal $u(t) = -Kx(t)$, where K is the optimal state feedback gain matrix, which is obtained by means of the algebraic equation of Riccati [7], it minimizes the $\mathcal{H}_2$-norm of the system. Alternatively, if a system with linear state-feedback is considered, the same result can be obtained when the energy of an auxiliary output signal $z_2(t)$, defined according to Fig. 1 (Cappelletti, [8]), is minimized.

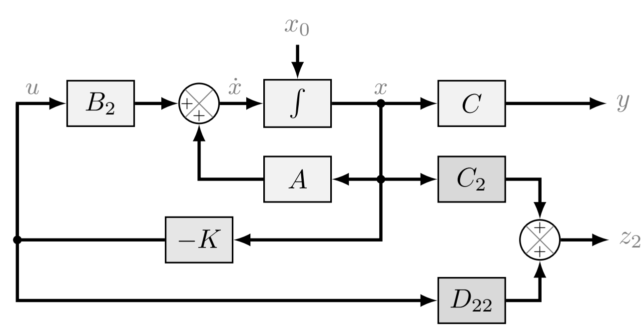


Fig. 1. State feedback with auxiliary output and disturbance of the initial state condition.

Notice that, based on the above mentioned figure,

$$\begin{array}{c} \dot{x}(t) = Ax(t) + x_0\delta(t) + B_2u(t), \\ z_2(t) = C_2x(t) + D_{22}u(t), \\ u(t) = -Kx(t), \\ y(t) = Cx(t). \end{array} \tag{1}$$

where x_0 is a state disturbance applied to the initial condition of the system.

If C_2 and D_{22} are chosen as orthonormal matrices, and they are defined as $C_2 = R_x^{1/2}$, and $D_{22} = R_u^{1/2}$, the spent energy of $z_2(t)$ signal

$$||z_2||_2^2 = \int_0^\infty (x'R_xx + u'R_uu)\ dt = J(x,u), \tag{2}$$

is minimal, if the following optimization problem is satisfied according to Scherer *et al.* [9].

$$\min_{Q,\ Y} \sigma \tag{3}$$

s.t.

$$\begin{pmatrix} 1 & x_0' \\ x_0 & Q \end{pmatrix} \succ 0, \qquad Q \succ 0, \tag{4}$$

$$\begin{pmatrix} QA' + AQ - Y'B_2' - B_2Y & Q & Y' \\ Q & -\sigma R_x^{-1} & 0 \\ Y & 0 & -\sigma R_u^{-1} \end{pmatrix} \prec 0, \tag{5}$$

where $K = YQ^{-1}$ is the optimal state-feedback gain matrix and σ a higher bound for $J(x,u)$.

Notice that, to the previous restrictions established to achieve an optimum gain matrix K, it is possible to add other ones, as process model uncertainties. Although, by increasing the number of restrictions, the feasible region is reduced, nevertheless the convexity keeps guaranteeing that the optimum found, if it exists, this one is unique and global.

B. Polytopic Formulation for Uncertain Systems

Consider the following model that represents a linear and time variant (LTV) system,

$$\begin{array}{l} \dot{x}(t) = A(t)\ x(t) + B_2(t)\ u(t) + x_0\delta(t), \\ [A(t) \quad B_2(t)] \in \Omega. \end{array} \tag{6}$$

Here Ω represents a set to which belongs the linear model family that describes the behavior of the system at different time instants.

If the set Ω is represented by a polytope written as:

$$\Omega = Co\{[A_1 \quad B_1], [A_2 \quad B_2], ..., [A_L \quad B_L]\}, \tag{7}$$

where Co means convex hull, and the pairs $[A_i \quad B_i]$ with $i = 1, 2, ...L$ are the vertices.

That is, a polytope is a polyhedron with L LTI models at its vertices. The real system represented in Eq. (6) is within the polytope, and although there exists infinite plants within it, each of them can be represented as a convex combination of its L vertex, as follows:

$$[A(t) \quad B(t)] = \textstyle\sum_{i=1}^L \lambda_i [A_i \quad B_i], \quad \text{where} \quad \sum_{i=1}^L \lambda_i = 1. \tag{8}$$

$L = 1$ corresponds to the nominal LTI model.

It is important to point out that the polytopical representation does not only consider the LTV systems, but also includes the linear systems with uncertainties in their parameters, and the non-linear systems, modeled by

$$\dot{x}(t) = f(x(t), u(t), t), \tag{9}$$

as long as its Jacobian matrices $[\frac{\delta f}{\delta x} \quad \frac{\delta f}{\delta u}]$ calculated in the different operation points, are within the uncertainties set Ω.

III. Robust Control

In this section, it is presented a robust regulator formulation that takes into account the model uncertainties in under polytopic representation as shown in the previous section.

In particular, the minimization of the classic nominal LQR objective function will be modified, to minimize an objective function that represents the worst case scenario.

Considering now the system described by Eq. (6) with an uncertainty associated with a set Ω, the minimization of the robust objective function is written as follows:

$$\min_{u(t)\ \in\ \mathbf{L}^2} \quad \max_{[A(t)\ B(t)]\ \in\ \Omega} \quad J(x,u), \tag{10}$$

where

$$J(x,u) = \int_0^\infty (x'(t)R_xx(t) + u'(t)R_uu(t))\ \ dt. \tag{11}$$

The maximization made on the set Ω corresponds to the choice of that plant $[A(t)\ B(t)] \in \Omega$ and $\forall t \geq 0$, such that if it is used as a representation model, it leads to the highest value of the objective function $J(x,u)$, known as the worst case. Notice that, the result of the minimization corresponds to the choice of that control action $u(t)$ of square integrable that minimizes the worst case.

Clearly, this is a min-max problem that although convex, is computationally very expensive [10]. As an alternative, following the same approach of the previous section, an upper bound for the robust objective function Eq. (11) will be looked for, and then, that upper bound will be minimized by using a control law by state-feedback via LMI.

A. Robust Control without Restrictions

Consider the system modeled by Eq. (6), where the set Ω is represented by the polytope (7). Each LTI model vertex of the polytope, is represented by

$$\begin{array}{c} \dot{x}(t) = A_ix(t) + x_0\delta(t) + B_{2i}u(t), \\ \text{with} \qquad i = 1, 2, ..., L, \end{array} \tag{12}$$

where an auxiliary output and a control action are defined as,

$$\begin{array}{c} z_2(t) = C_2x(t) + D_{22}u(t), \\ u(t) = -Kx(t), \end{array} \tag{13}$$

as in the nominal case of the subsection II-A, but now applied to each vertex.

If an upper bound is chosen for the objective functions of all LTI models that conform the convex hull of the uncertainty polytope, then any plant can be represented from a convex combination of them. Therefore, following the development of the subsection II-A, the problem of robust minimization is posed as:

$$\min_{\sigma,Y,Q\succ 0} \sigma \tag{14}$$

s.t.

$$\begin{pmatrix} 1 & x_0' \\ x_0 & Q \end{pmatrix} \succ 0,$$

$$\begin{pmatrix} QA_i' + A_iQ - Y'B_{2i}' - B_{2i}Y & Q & Y' \\ Q & -\sigma R_x^{-1} & 0 \\ Y & 0 & -\sigma R_u^{-1} \end{pmatrix} \prec 0,$$
$$\text{with} \qquad i = 1, 2, ..., L. \tag{15}$$

obtaining the robust feedback gain matrix

$$K_{opt} = Y_{opt} Q_{opt}^{-1}. \tag{16}$$

1) Robust Asymptotic Stability: Next, the lemma of the invariant ellipsoid according to Kotare *et al.* [10] will be used to show that the robust control law obtained as a solution to the minimization problem (14), stabilizes asymptotically the system with uncertainties.

Lemma Consider the system (6) and a set of associated uncertainties Ω. Let Ω be a polytope described by Eq. (7). Suppose that there exists at the instant $t = 0$, $Q^{-1} \succ 0$, $\sigma > 0$ and $K = YQ^{-1}$ such that (15) satisfied. Suppose also that $u(t) = Kx(t)$. So if $t = 0$, it is satisfied

$$x_0' \; Q^{-1} x_0 < 1, \quad \text{then} \tag{17}$$

$$\max_{[A(t) \;\; B(t)] \,\in\, \Omega, \; \forall t \geq 0} \quad x(t)' Q^{-1} x(t) < 1, \;\; \forall t \geq 0. \tag{18}$$

That is, the inequality (17) is an invariant ellipsoid for all the states of the uncertain system.

Then, it will be shown that each vertex of the polytope (7) satisfies Eq. (18).

Defining $A_{cli} = A_i - B_{2i}K$ and $C_{cl} = C_2 - KD_{22}$, and using the Schur complement, the second LMI in (15) is equivalent to the following matrix inequality:

$$A_{cli}' Q^{-1} + Q^{-1} A_{cli} \prec -\tfrac{C_{cl}' C_{cl}}{\sigma}. \tag{19}$$

Then, pre and post multiplying the inequality (19) by $x_i'(t)$ and $x_i(t)$ respectively, it is arrived at,

$$\underbrace{x_i'(t) A_{cli}'}_{\dot{x_i}'(t)} Q^{-1} x_i(t) + x_i'(t) Q^{-1} \underbrace{A_{cli} x_i(t)}_{\dot{x_i}(t)} < -\tfrac{x_i'(t) C_{cl}' C_{cl} x_i(t)}{\sigma},$$
$$\dot{x_i}'(t) Q^{-1} x_i(t) + x_i'(t) Q^{-1} \dot{x}_i(t) < -\tfrac{(x_i'(t) R_x x_i(t) + u_i'(t) R_u u_i(t))}{\sigma}. \tag{20}$$

Integrating between $t = 0$ and $t^* \in (0, \infty)$, results

$$x_i'(t) Q^{-1} x_i(t) \,|_0^{t^*} < -\tfrac{1}{\sigma} \underbrace{\int_0^{t^*} (x_i'(t) R_x x_i(t) + u_i'(t) R_u u_i(t)) dt}_{(>0, \; \forall x_i(t) \neq 0)},$$

$$x_i'(t^*) Q^{-1} x_i(t^*) - x_0' Q^{-1} x_0 < 0, \quad \forall i = 1, 2, ..., L. \tag{21}$$

Therefore, for any system within the polytope Ω and $\forall t > 0$, one has

$$x'(t) Q^{-1} x(t) < x_0' Q^{-1} x_0 < 1. \tag{22}$$

The function $V(x(t)) = x'(t) Q^{-1} x(t)$, is a Lyapunov function for all systems represented by the model (6) feedbacked by $u(t)$, it is easy to see that this function decreases as time increases, which indicates the asymptotic stability of the feedback system, since $x(t) \to 0$ for $t \to \infty$.

B. Robust control with restriction in the manipulated variable

In the subsection III-A the minimization problem of a robust objective function without restrictions was formulated, and an upper bound σ was fixed for it. This subsection shows how to incorporate in this problem a restriction on the amplitude of $u(t)$ including an LMI.

Being $K = YQ^{-1}$ then,

$$|u(t)|^2 = x(t)' Q^{-1} Y' Y Q^{-1} x(t). \tag{23}$$

Defining

$$\begin{aligned} v(t) &= Q^{-1/2} x(t), \\ H &= YQ^{-1/2}, \end{aligned} \tag{24}$$

Notice that, the Eq. (18) can be written as

$$|u(t)|^2 = v(t)' H' H \; v(t), \tag{25}$$

where the matrix $H'H$ is a symmetric matrix, and therefore it can be diagonalized orthogonally as

$$H'H = T' \Lambda T. \tag{26}$$

Defining

$$q(t) = Tv(t), \tag{27}$$

and being n the number of states, we have

$$|u(t)|^2 = q(t)' \Lambda \; q(t) = \sum_{k=1}^{n} \lambda_k \; q_k^2(t), \tag{28}$$

where λ_k, with $k = 1, ..., n$, are the singular values of H, so that the following inequality is verified:

$$|u(t)|^2 \leq \lambda_{max}(H'H) \;\; |q(t)|^2. \tag{29}$$

Taking into account that the T matrix only produces one rotation,

$$|q(t)|^2 = |v(t)|^2 = x(t)' Q^{-1} x(t), \tag{30}$$

and that $\lambda_k(H'H) = \lambda_k(HH')$, the Eq. (29) can be rewritten as:

$$|u(t)|^2 \leq \lambda_{max}(YQ^{-1}Y')(x(t)' Q^{-1} x(t)). \tag{31}$$

Since $V(x(t)) = x(t)' Q^{-1} x(t)$ has a maximum given by the initial time

$$x(t) Q^{-1} x(t) \leq x_0' Q^{-1} x_0 < 1, \tag{32}$$

then, setting a bound for the maximum amplitude of the manipulated variable, it can be written

$$|u(t)|^2 < \lambda_{max}(YQ^{-1}Y') < U_{max}^2. \tag{33}$$

So that, if it is satisfied

$$\lambda_{max}(YQ^{-1}Y') < U_{max}^2, \tag{34}$$

it is guaranteed that

$$|u(t)| < U_{max} \quad \forall t. \tag{35}$$

Finally, using the Schur complement the inequality (34) can be rewritten as

$$\begin{pmatrix} U_{max}^2 & Y \\ Y' & Q \end{pmatrix} \succ 0. \tag{36}$$

The LMI (36) is a new convex region that intersects with the convex regions represented by the LMIs (15), and the found gain matrix value $K_{opt} = Y_{opt} Q_{opt}^{-1}$, produces $u_{opt}(t)$ that stabilizes the uncertain system (6), and its amplitude does not exceed U_{max} .

However, this is a conservative approach since for a system with uncertainties, even without restrictions and with an objective function integrated in an infinite interval, the matrix K_{opt} is not necessarily constant.

C. Obtaining a variable feedback gain matrix K_K by recalculating its value at different time intervals

One way to increase the response speed without compromising the stability and restrictions imposed on the original problem is to recalculate the controller gain matrix at different times instead of using a single gain for the whole regulation process.

To demonstrate that this methodology maintains the robust stability of the feedback system, consider the minimization problem (14), with the constraints (15) and (36).

According to the invariant ellipsoid lemma [11], if the optimization problem is feasible at $t = 0$, it will also be at some later time $t = T_M$, where T_M is some given time interval. That is to say, if the states of the system are measured at that moment, the restriction has to be

$$x'(t_M)Q^{-1}x(t_M) < x_0'Q^{-1}x_0 < 1, \tag{37}$$

or equivalently, the LMI

$$\begin{pmatrix} 1 & x'(T_M) \\ x(T_M) & Q \end{pmatrix} \succ 0, \tag{38}$$

it is feasible. Furthermore, this is the only LMI that explicitly depends on the states measurement in the minimization problem.

Therefore, taking the state $x(T_M)$ as the initial condition, now the following optimization problem is posed:

$$\min_{\sigma,Y,Q\succ 0} \sigma \tag{39}$$

s.t.

$$\begin{gathered} \begin{pmatrix} 1 & x'(T_M) \\ x(T_M) & Q \end{pmatrix} \succ 0, \\ \begin{pmatrix} QA_i' + A_iQ - Y'B_{2i}' - B_{2i}Y & Q & Y' \\ Q & -R_x^{-1} & 0 \\ Y & 0 & -R_u^{-1} \end{pmatrix} \prec 0, \\ \text{with} \qquad i = 1,2,...,L. \\ \begin{pmatrix} U_{max}^2 & Y \\ Y' & Q \end{pmatrix} \succ 0. \end{gathered} \tag{40}$$

The solution to this problem are the optimal values of Q and Y, which will be defined as Q_{T_M} and Y_{T_M} respectively, which due to the model uncertainty and the constraints imposed on the manipulated variable, it will probably have different values from the optimal values (Q and Y) obtained from the minimization problem with initial ondition x_0.

Similar to what was stated before, the value of the gain $K_{T_M} = Y_{T_M}Q_{T_M}^{-1}$, minimizes the robust objective function (10), which is now defined as $J_{T_M}(x,u)$, and its minimum value will be:

$$J_{T_M}(x_{opt}, u_{opt}) = x'(T_M)\ P_{T_M}\ x(T_M) < \sigma. \tag{41}$$

The key here, is to note that

$$x'(T_M)\ Q_{T_M}^{-1}\ x(T_M) \le x'(t_M)\ Q^{-1}\ x(T_M), \tag{42}$$

due to Q_{TM}^{-1} is optimal, while Q^{-1} is only feasible at time $t = T_M$ [10]. Therefore, and using the inequality (37), we have

$$\underbrace{x'(0)\ Q^{-1}\ x(0)}_{\text{optimal } t\,=\,0} > \underbrace{x'(T_M)\ Q^{-1}\ x(T_M)}_{\text{feasible in } t\,=\,T_M} \ge \underbrace{x'(T_M)\ Q_{T_M}^{-1} x(T_M)}_{\text{optimal in } t\,=\,T_M}. \tag{43}$$

Now, the previous development made between the instants of time $t = 0$ y $t = T_M$, can be repeated between the time's instants $t = kT_M$ and $t = (k+1)T_M$ with $k = 1,2,...,\infty$.

Without loss of generality doing $T_M = 1$, and defining $x(kT_M) = x(k) = x_k$, it can be written,

$$\begin{gathered} x_{k+1}'\ Q_{k+1}^{-1}\ x_{k+1} < x_k'\ Q_k^{-1}\ x_k, \\ \forall k = 0,1,2,...,\infty. \end{gathered} \tag{44}$$

Notice that, in every moment of time $t = kT_M$ a convex optimization problem is solved, so every minimum ($x_k'\ Q_k^{-1}\ x_k$) obtained, it is unique and corresponds to the optimal solution for that moment.

Therefore, $V(x_k) = x_k'\ Q_k^{-1}\ x_k$, with $k = 1,2,...,\infty$, it is a strictly decreasing Lyapunov function for the uncertain system (6) with restrictions. Being $\sigma V(x_k)$ the upper bound of the robust objective function (10), which is reduced at each recalculation instant.

IV. Example. Regulation of a CSTR

In this section, the tuning technique here presented is applied to a CSTR proposed by Morningred *et al.* [12], where the reactant A becomes the product B by means of an exotermic chemical reaction, modeled by the following equations:

$$\begin{aligned} \dot{C}_A(t) =& \frac{q_e}{V}\left(C_{Ae} - C_A(t)\right) - k_0 e^{\frac{-E_R}{T(t)}} C_A(t) \\ \dot{T}(t) =& \frac{q_e}{V}\left(T_e - T(t)\right) + k_1 e^{\frac{-E_R}{T(t)}} C_A(t) \\ &+ \frac{q_c(t)}{V}\left(1 - e^{\frac{-k_3}{q_c(t)}}\right)\left(T_{ce} - T(t)\right) \end{aligned} \tag{45}$$

The parameters of the model are reported by Morningred and co-workers [12].

The reaction takes place in a stirred cylindrical tank, as shown in Fig. 2. Also, the CSTR operates at constant volume.

Furthermore, the reactant concentration C_A and the reactor temperature T are considered measured variables. This reactant concentration is indirectly controlled, manipulating by means of the single coolant stream $q_c(t)$ that circulates through a serpentine.

1) Design Objectives: The proposed objectives controller design for this problem are,

- to guarantee the stability of the non-linear system,
- to minimize both measurement and process noises,
- to satisfy an amplitude restriction in the manipulated stream whose operating range is

$$85 \le q_c(t) \le 116 \quad \text{L min}^{-1}. \tag{46}$$

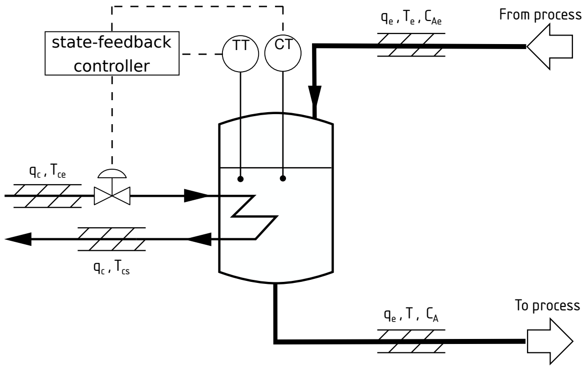


Fig. 2. An illustrative diagram of the CSTR control.

Therefore, to obtain the variable feedback gain matrix K_k, that satisfies the above objetives, the optimization problem (39) subject to (40) is solved using the LMI Lab [13] package of MATLAB software.

Fig. 3a shows the reaction curve and the energy dissipation line. In addition, three points are included on the reaction curve. There, the blue dot indicates the stationary point of the system for a coolant stream $q_c = 111.72$ L min^{-1}, which corresponds to a concentration $C_A = 0.14$ mol L^{-1} and a temperature $T = 431.32$ K.

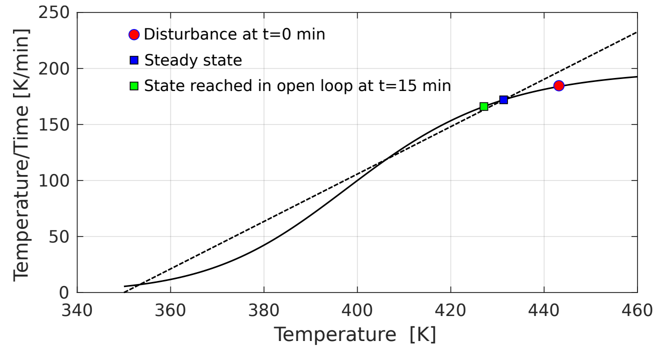
In order to analyze the performance of the control system with the proposed regulator, a disturbance is introduced, leading the states to the following values: $C_A = 0.08$ mol L^{-1} and $T = 443.16$ K. This perturbed state vector is represented on the reaction curve by the red circle. Three simulations are carried out, the first one is in open loop, the second one implements a controller with a constant feedback gain and, the third one includes a variable feedback gain matrix. Also, for the last case, the gain matrix is updated every 6 seconds. The evolution of these three systems is presented in the states diagram of Fig. 3b.

Fig. 4 shows the response time of the states for the three systems, in Fig. 4a the reactant concentration and in Fig. 4b the reactor temperature is represented.

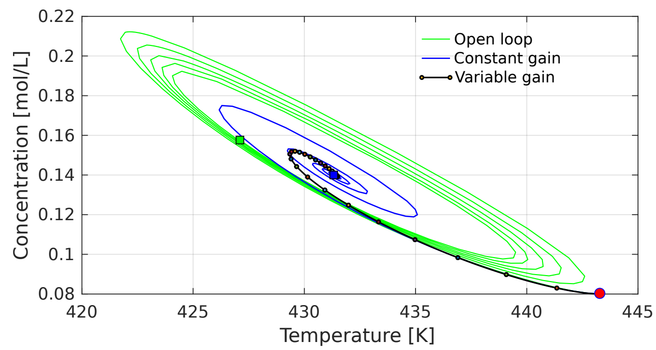
Fig. 5 shows the manipulated variable. This variable has a restriction on its amplitude $|\Delta q_c| < 4$, the maximum level was chosen by making a difference between the maximum available flow and the operating flow for the equilibrium point. The Fig. 5a shows the increment $\Delta q_c(t)$, while the Fig. 5b shows the full range of $q_c(t)$.

Also, notice that dynamic responses have a discontinuity both in the flow increment and in the manipulated flow. This behavior is due to the recalculation of the gain matrix, at this moment a new optimization problem with a new initial condition is solved. In addition, it is noteworthy that although the gain is constant during the interval, the manipulated flow varies as the states change, something that does not happen in the traditional discrete control.

Fig. 6 shows the gains calculated via LMI, for the design of the controllers with constant gain matrix and variable gain matrix.

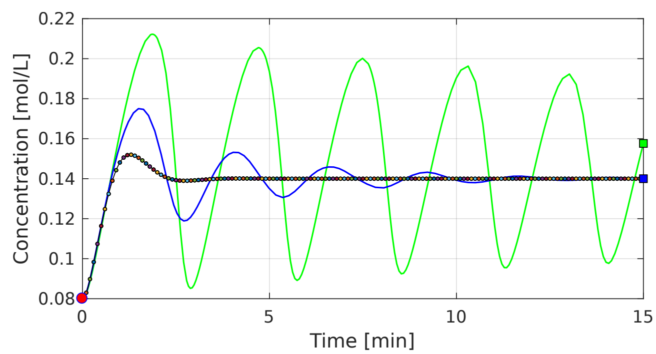


(a) Reaction curve and dissipation line for $q_c = 111.72$ L min^{-1}

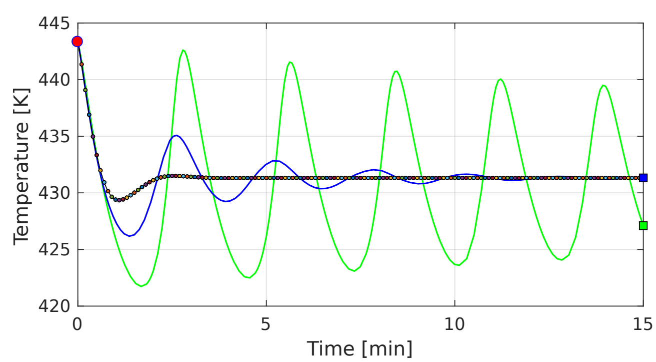


(b) State diagram for the three systems

Fig. 3.



(a) Concentration dynamic response for the three systems

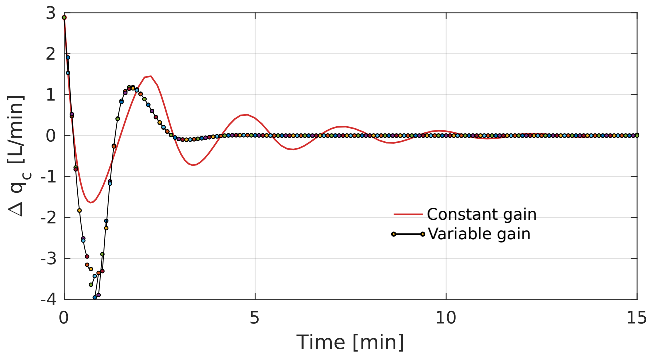


(b) Temperature dynamic response for the three systems

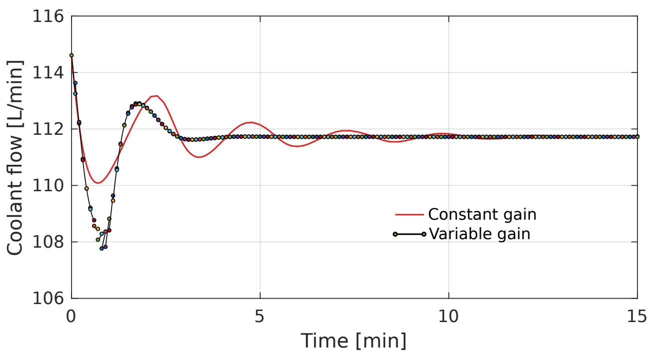
Fig. 4.

V. Conclusion

In this paper, it is presented a robust regulator design with variable gain matrix for a system with a polytopic uncertainty

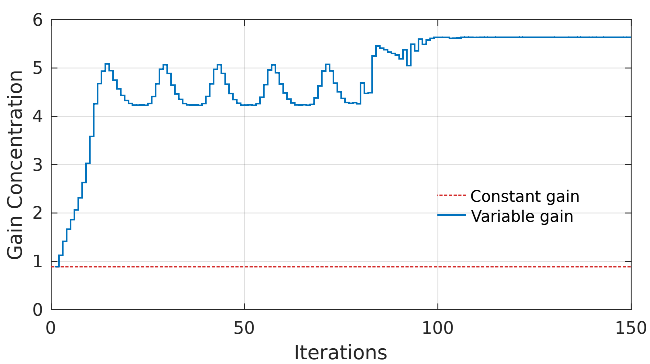


(a) Manipulated flow increment dynamic response by the two controllers in deviation variable

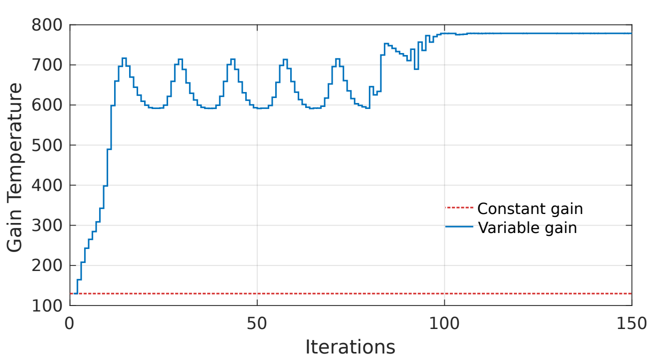


(b) Manipulated flow dynamic response by the two controllers

Fig. 5.



(a) Feedback gain for concentration.



(b) Feedback gain for temperature.

Fig. 6. Variations of the gain matrix recalculated every 6 seconds.

representation, being able of satisfying multiple objectives, through LMIs. It should be noted that, the main proposal in this paper is that the regulator makes use of an LTV model under continuous time representation of the process. Consistent with this, both the linearization of the non-linear model for the construction of the polytope and the LMIs are done in continuous time, and only the optimal gain matrix is recalculated, at pre-established time intervals. The main advantage of this, is that the proposed regulator allows to satisfy simultaneously with multiple objectives such as asymptotic stability, minimization of the standard $\mathcal{H}_2$, robustness and restriction of a bound imposed on the manipulated variable, with a significant reduction of conservatism.

As an example, a typical application of chemical engineering was chosen, as the chemical reactor. The simplicity of the chosen problem aims to highlight, in this work, the benefits of the proposed regulator along with its dynamic characteristics. The comparison of the controllers with constant and variable gain matrix shows a significant improvement in settling time for this last case (at least three times faster), and this is due to a better use of the coolant stream.

Finally, it should be noted that this proposal can be implemented to a discrete system, adapting the LMIs under this formulation. Also notice that, the recalculation of the gain matrix is independent of the sampling period of the system, and does not necessarily have to coincide.

References

[1] K. J. Aström and R. H. Murray, *Feedback Systems. An Introduction for Scientists and Engineers*. Princeton University Press, 2008.

[2] D. E. Seborg, T. F. Edgar, D. A. Mellichamp, and F. J. D. III, *Process Dynamic and Control*. John Wiley & Sons, 2011.

[3] E. J. Adam, *Instrumentacion y Control de Procesos*. Ediciones UNL, 2014.

[4] Y. Cao and K. Fang, "Parameter-dependent lyapunov function approach to stability analysis and design for polytopic systems with input saturation," *Asian Journal of Control*, vol. 9, no. 1, pp. 1–10, 2007.

[5] D. Henrion, S. Tarbouriech, and G. Garcia, "Output feedback robust stabilization of uncertain linear systems with saturating controls: an lmi approach," *IEEE Transactions on Automatic Control*, vol. 44, 1999.

[6] D. Henrion and S. Tarbouriech, "LMI relaxations for robust stability of linear systems with saturating controls," *Automatica*, vol. 35, pp. 1599–1604, September 1999.

[7] C. N. Rautenberg and C. E. D'attellis, *Control Lineal Avanzado y Control Óptimo*. Asociación Argentina de Control Automático (AADECA), 2004.

[8] C. A. Cappelletti, "Control Óptimo de Procesos Industriales Utilizando Desigualdades Lineales Matriciales," Master's thesis, Universidad Nacional del Litoral, 2016.

[9] C. Scherer, P. G. A., and M. Chilali, "Multiobjetivo output-feedback control via lmi optimization," *IEEE Transactions on Automatic Control*, vol. 42, no. 896–911, 1997.

[10] M. Khotare, V. Balakrishman, and M. Morari, "Robust constrained model predictive control using linear matrix inequalities," *Automatica*, vol. 32, no. 10, pp. 1361–1379, 1996.

[11] S. Boyd, L. E. Ghaoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*. Society for Industrial and Applied Mathematics (SIAM), 1994.

[12] J. D. Morningred, B. E. Paden, D. E. Seborg, and D. A. Mellichamp, "An adaptive nonlinear predictive controller," *in Proc. Amer. Contr.Conf.*, vol. 2, pp. 1614–1619, 1990.

[13] P. Gahinet, A. Nemirovski, A. J. Laub, and M. Chilali, *LMI Control Toolbox User's Guide*. MathWorks, 05 1995.